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# STATISTICAL TOLERANCE ANALYSIS OF MECHANISMS WITH INTERACTIONS BETWEEN DEVIATIONS – A METHODOLOGY WITH 10 EASY STEPS

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## ABSTRACT

The time-dependent motion behavior of a mechanism is essentially affected by different kinds of deviations. Consequently, the product developer has to analyze the mechanism and its kinematic behavior as early as possible to ensure the product's functional capabilities. However, possible interactions between the deviations and their effects on the system's motion, and thus the functionality are not considered yet.

This paper presents a methodology, consisting of 10 easy steps, which enables the product developer to perform a statistical tolerance analysis of a system in motion, which underlies different kinds of deviations as well as several interactions among them. Therefore, the identification as well as the determination (statistically) of the interactions is required, which can be done using numerical simulations like multi-body-dynamics or manufacturing process simulations. An appropriate mathematical representation of the interactions is done using meta-models (like Artificial Neural Networks). These can be easily integrated into the tolerance analysis' functional relation. A case study of a non-ideal cross-arm window regulator illustrates the methodology's practical use.

*Keywords: statistical tolerance analysis, artificial neural networks, robust design, simulation, design methods*

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# 1 MOTIVATION

“You can’t always get what you want” – this title of a Rolling Stones song describes very clearly that each individual’s wishes and needs will not be entirely satisfied – there are always uncertainties in our daily living. Consequently, this quote can be seen in several contexts of daily living – including product development.

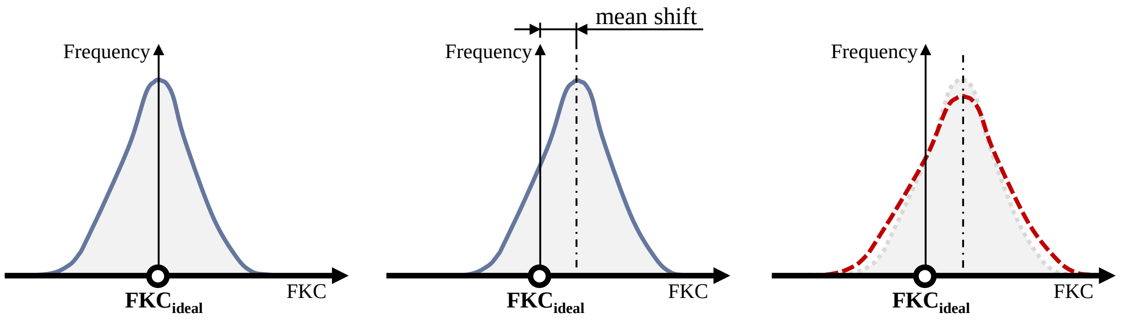
In today’s product development the product developer usually designs products and its components by means of computer-aided design tools (CAD). However, these tools do not represent the entire reality, since both the dimensions and the shape of the parts are considered ideal, without any deviations. Hence, deviations and their effects on a product are quite often not taken into account during product development – or at least not considered until problems appear.

Dimensional and geometrical deviations can appear in every stage of the product’s lifecycle: manufacture, assembly and in the product’s use (Walter et al., 2012). These deviations can be traced back to manufacturing discrepancies as well as assembly imprecision. Moreover, operation-dependent deviations (such as the deformation of parts due to inertial forces) also appear during the product’s use. These deviations affect the product’s functionality which depends largely on the interaction of its components and their geometries. The functional capabilities of a product are usually defined by appropriate characteristics – the so-called functional key characteristics (FKCs) (Thornton, 1999). For instance the gap between the two moving parts of a combustion engine, which ensures the collision-free motion of the parts, can be considered a FKC.

In general, the different kinds of deviations can be classified into two main types, depending on the effects on the FKCs (Walter et al., 2013):

- Random deviations resulting in a variation of the FKC (e.g. manufacturing-caused variations of dimensions such as height and length) (Figure 1a).
- Systematic deviations (like an operation-dependent deformation of a part) causing a mean shift of the FKC’s distribution (Figure 1b).

Aside from these (direct) effects, effects from the deviations can also appear among themselves. These so-called interactions cause an additional variation of the FKC (Figure 1c).




(a) Variation of the FKC (due to random deviations)

(b) Variation (random deviations) and mean shift (systematic deviations) of the FKC

(c) Additional variation of the FKC (due to interactions between deviations)

*Figure 1. Effects of random and systematic deviations as well as interactions on a FKC*

Usually statistical tolerance analyses are used to investigate how the appearing deviations affect a product’s FKCs. However, possible interactions between the deviations as well as the resulting effects among themselves as well as on the FKC are not yet considered (Walter et al., 2012). Moreover, this issue becomes much more complex should the tolerance analysis be used to investigate a time-dependent system in motion, whose parts are subject to random and systematic deviations (Stuppy and Meerkamm, 2009).

This paper focuses on the statistical tolerance analysis of systems in motion. In this context, a methodology is presented which enables the product developer to gain information about the effects of time-dependent deviations (from different stages of the product lifecycle) on a mechanism’s FKCs. Furthermore, the interactions between these deviations as well as their corresponding time-dependent effects on the FKCs can be taken into account.

Therefore, the current state of the art concerning the tolerance analysis of mechanisms is discussed in the upcoming section 2. Since meta-models are used to represent the interactions between deviations, the use of meta-modeling techniques in tolerance management- and robust design-related publications is also detailed in this section. The tolerance analysis methodology is presented in section 3. In order to show the methodology's practical use and to concisely detail each individual step, a statistical tolerance analysis of a car's window regulator mechanism is performed. The mechanism's components are subject to both random and systematic deviations, which affect the window's position during the motion sequence – the closing of the window (section 4). The 10 steps of this case study's statistical tolerance analysis are detailed in the sub-sections of section 5.

# 2 STATE OF THE ART

In product development today the two major objectives of dimensional management – tolerance analysis and tolerance synthesis – are well known and widely used. However, existing tolerance analysis approaches do not integrate the specific aspects of time-dependent systems in motion. In this context the different kinds of time-dependent deviations and the interactions entailed between these deviations, as well as their effects on FKCs, have to be emphasized.

In 1957, Morrison (1957) stated that interactions between varying parameters are an essential aspect in reducing the variation in FKCs. Despite this potential, the interactions between deviations have not drawn much attention in tolerance management (Hasenkamp et al., 2009). This prompted Hasenkamp et al. (2009) to identify, when considering the entire product lifecycle, that the development of integrated methods is a promising as well as needed aim of tolerance management and robust design.

## 2.1 Statistical tolerance analysis of mechanisms

The existing publications on the tolerance analysis of mechanisms can be divided into three groups, depending on the kind(s) of deviations taken into account. First, several publications consider the effects of manufacturing-caused deviations on the kinematic behavior of a mechanism – including systems with both lower (e.g. Adabi et al., 2010) and higher kinematic pairs (Bruyere et al., 2007). Secondly, operation-dependent deviations appear during the mechanism's use and affect the system's motion. Sacks and Joskowicz (1998) consider the operation-dependent displacement of components due to joint clearance, while the deformations of parts due to the forces, resulting from the system's motion, are taken into account by Dupac (2010) and Imani and Pour (2009). Finally, manufacturing-caused as well as operation-dependent deviations are integrated into the tolerance analysis of a mechanism (e.g. Hanzaki et al., 2009). However, the time-dependencies of a mechanism and the appearing deviations are not taken into account in the previously mentioned works.

Huang and Zhang (2010) present a robust design approach, which enables the product developer to analyze a system in motion with manufacturing-caused deviations as well as imperfect joints. Furthermore, the "integrated tolerance analysis of systems in motion" is introduced by Stuppy and Meerkamm (2009). This approach allows the statistical tolerance analysis of a mechanism with both manufacturing-caused and operation-dependent deviations (deformation and displacement due to joint clearance). Moreover, the "integrated tolerance analysis" is not only limited to Gaussian distributions of the appearing deviations (such as in Huang and Zhang's approach). Wartzack et al. (2011) detail an appropriate result visualization of an "integrated tolerance analysis of a system in motion".

In summary, the listed publications focus on the effects of manufacturing and/or operation-depending deviations on the FKCs of a technical system. Possible interactions between the different deviations as well as the resulting effects among themselves and on the FKCs have not yet been taken into account.

## 2.1 Use of meta-models in tolerance management and robust design

In 2002, Hong and Chang (2002) stated that meta-models like Artificial Neural Networks (ANNs) could pave the way to "a systematic method which automates this procedure, incorporating the domain specific knowledge as well as the geometry and process knowledge". The use of meta-modeling techniques is currently still limited in tolerance-related work – despite their auspicious potential (Dantan et al., 2012). However, an extended use of meta-models in tolerance management in recent years is commented upon by Dantan et al. (2012).

The use of meta-modeling techniques in tolerance-related issues can be separated into two main fields of application: On the one hand, tolerance-cost-relations (describing the dependencies between tolerances and the corresponding manufacturing costs) can be replaced by appropriate meta-models. These relations are essential for tolerance synthesis, but are usually unknown. Consequently, several meta-modeling techniques can be applied to approximate these tolerance-cost-relations, e.g. Response

Surface Methodology (RSM) (Kim and Cho, 2001) and ANNs (Chen, 2001). On the other hand, the relations between a system's FKC and the appearing deviations can be formulated by means of meta-models. These relations are required to perform statistical tolerance analysis. For instance, Schleich et al. (2012) use the RSM to approximate the deformation of a beam in bending, which underlies geometrical deviations. Furthermore, ANNs are applied in the context of tolerance analyses. Andolfatto et al. (2012) trained an ANN that predicts the effects of manufacturing-caused deviations on a system's assembly behavior.

## 3 METHODOLOGY: TOLERANCE ANALYSIS IN 10 EASY STEPS

According to Salomons et al. (1998), the tolerance process during product development involves three essential activities: First, the appearing deviations must be defined and limited by tolerances (tolerance specification). The effects of these tolerances on the considered system's varying FKCs are investigated in the second step, the tolerance analysis. Finally, the previously-defined tolerances are modified based on the results of the tolerance analysis (tolerance synthesis).
The holistic methodology presented incorporates these three activities. Furthermore, it includes an extension of the existing "integrated tolerance analysis of systems in motion" in order to take into account interactions between deviations. The methodology with its ten steps is shown in Figure 2.

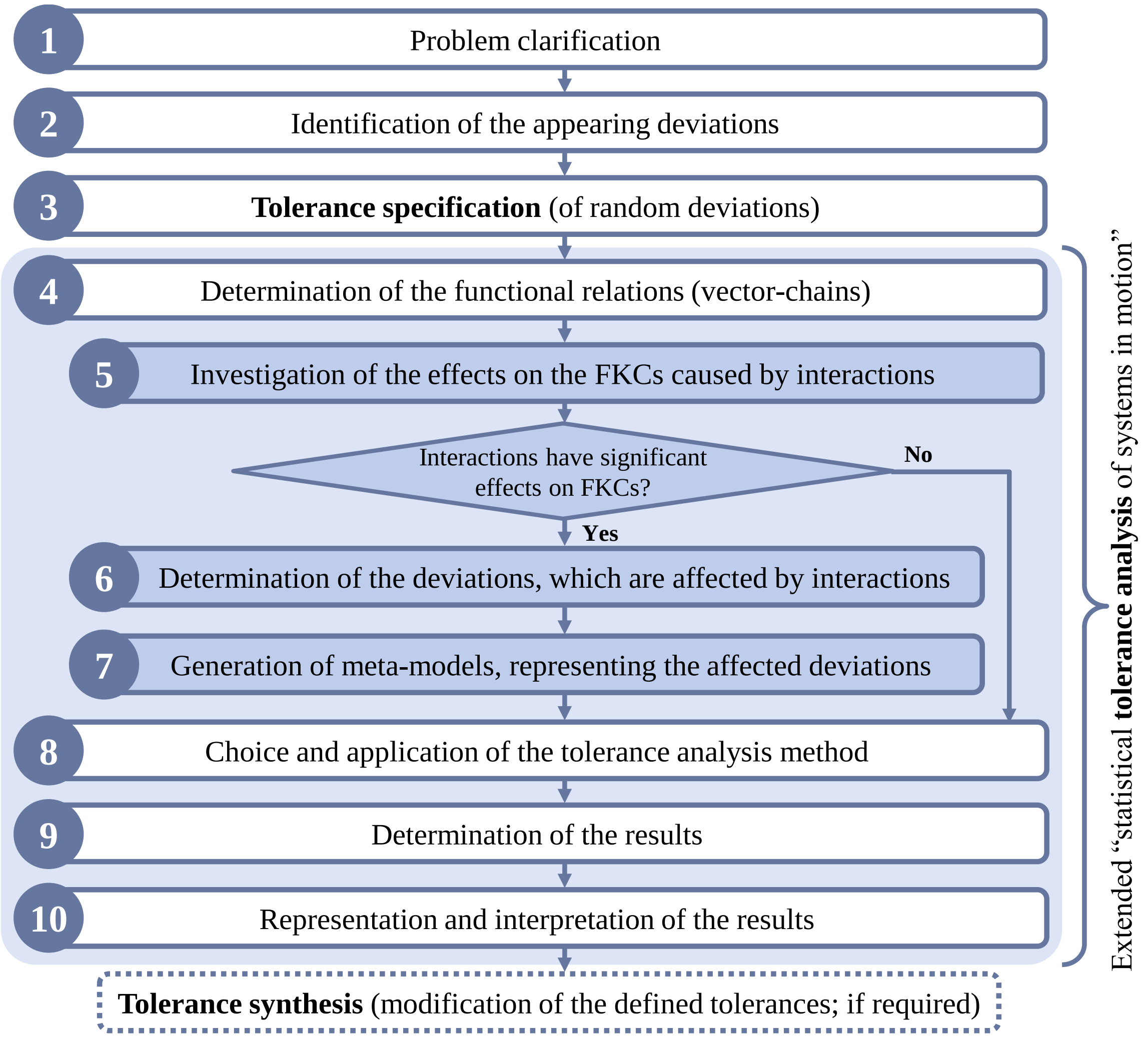



*Figure 2. Methodology: Statistical tolerance analysis of systems in motion with interactions between the appearing deviations*

The first and thus the most important step is the clarification of the considered tolerance-related problem. This goes hand in hand with the identification of the system's FKCs (using e.g. FMEA), since those should be used to quantify the functional capabilities of the mechanism during its use. Furthermore, the so-called upper and lower specifications limits (USL and LSL) must also be defined.

A FKC within those (usually time-dependent) limits implies that the mechanism's functional capabilities are ensured. The second step focuses on the appearing deviations. According to Figure 1, random as well as systematic deviations must be identified. Consequently, the presence of interactions between these deviations can be derived. Afterwards, appropriate tolerances must be defined, to limit the random manufacturing-caused deviations. Therefore, the definition of the tolerance type (e.g. parallelism) as well as the corresponding tolerance value is required for each random deviation (step 3). Consequently, the information needed for the tolerance analysis of the considered non-ideal mechanism is defined.
A tolerance analysis consists of three main steps (Stuppy and Meerkamm, 2009): First, mathematical relations are needed that describe the dependencies of the mechanism's FKCs and the appearing deviations (step 4). Afterwards, a destined number of virtual mechanisms is generated (based on the chosen tolerance analysis method; usually > 10,000) and the FKCs are determined for each of these virtual mechanisms in step 8. The determination and representation of the results of the tolerance analysis (steps 9 and 10) complete the tolerance analysis.
However, the consideration of interactions between deviations requires a modification of the common tolerance analysis procedure, in particular step 4 – the determination of the functional relation. Since these relations do not include the effects of interactions, additional steps are needed. These steps 5–7 (blue boxes in Figure 2) incorporate the determination of the deviations, which underlie interactions as well as their representation by means of appropriate meta-models. These meta-models, which now quantify the effects of interactions, can be easily integrated into the functional relations (step 4).
Consequently, the product developer can establish functional relations, which take into account the effects of deviations as well as the effects of interactions on a mechanism's FKCs. Finally, the previously-defined tolerances have to be changed (tolerance synthesis), if the investigated tolerance specification was identified to cause functional problems.

## 4 DEMONSTRATOR: CAR CROSS-ARM WINDOW REGULATOR

To detail each step of the methodology, its practical use is shown in a case study of a car's cross-arm window regulator (Figure 3). The window lifter's parts are subject to both manufacturing-caused and operation-dependent deviations (deformation of the inner arm). The geometries of the two lifter arms and thus the mechanism's motion are defined by dimensions $l_1$, $l_2$, $l_3$ and $l_4$ as well as the angle α.

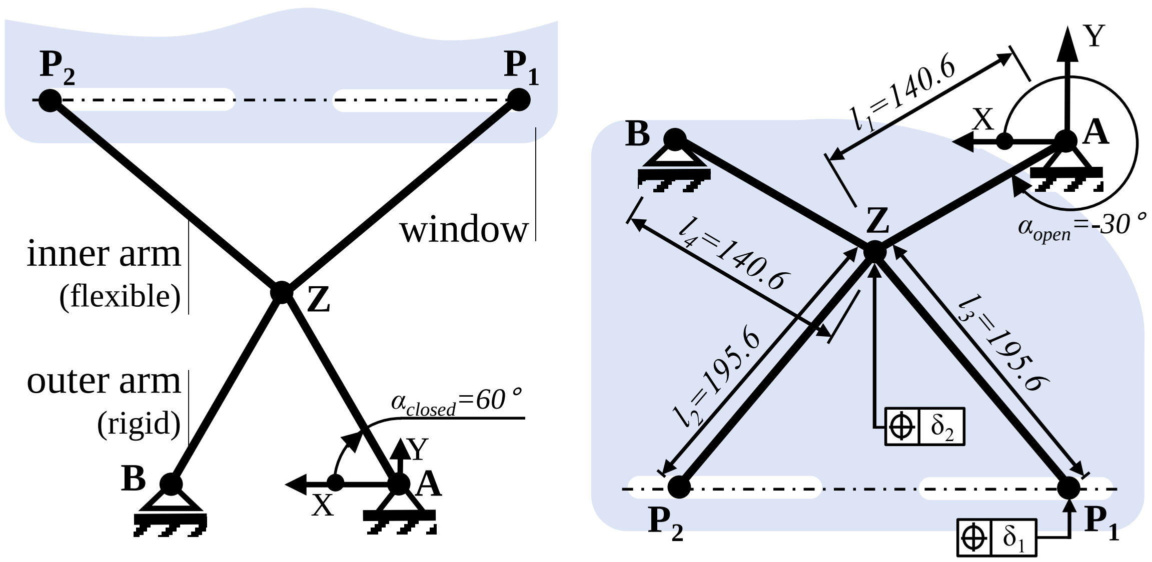



*Figure 3. Cross-arm window regulator: (left) closed window (right) opened window*

The mechanism is used to close the car's window within three seconds – starting at the lowest position (α = -30 °). Figures 3 and 4 clarify that the dimensional and geometrical deviations of the regulator's components result in a height difference Δh between the two joints of the arms (joints $P_1$ and $P_2$), causing an inclination of the window during the motion sequence.

## 5 THE 10 EASY STEPS IN DETAIL

### 5.1 Step 1: Clarification the considered problem

During its use, the window regulator has to ensure the failure-free motion of the window (no tilt of the window in its side guides). Moreover, a sufficient contact area (overlap) of the window within the car door's upper weatherstrip is required to avoid/reduce disruptive wind noise in the car's interior space. Both requirements are essentially affected by the window's inclination. Consequently, the regulator's FKC is the inclination angle φ, which results from the height difference Δh between the window's points of support $P_1$ and $P_2$ (Figure 4).

To fulfill the given functional requirements, the FKC has to be limited. In this case the inclination angle φ should not exceed an error of ±0.1° at any point in time (and especially in the window's upper position) of the mechanism's motion. Consequently, the upper (USL = +0.1°) and lower specification limit (LSL = -0.1°) of the FKC are defined.

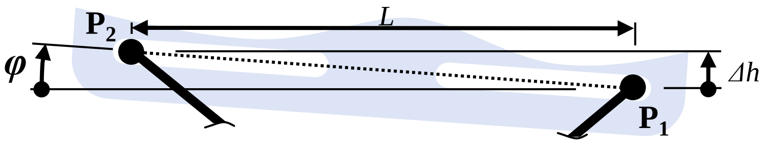



*Figure 4. Functional Requirement: Limitation of the window's inclination angle φ*

### 5.2 Step 2: Identification of the appearing deviations

Based on the definition of the FKC, the appearing deviations of the window regulator (which affect the FKC φ) as well as the resulting interactions between these deviations must be identified.

The outer arm (material: steel) underlies a deviation in its length $l_3$ = 195.6 ± 0.1 mm. Moreover, two position deviations $\delta_1$ and $\delta_2$ of the joint's axis (joints in $P_1$ and Z) appear (Figure 3). Since the inner arm is made from a thermoplastic material (using injection molding), shrinkage as well as warpage of the arm occur. This manufacturing-caused deviation of the arm from its nominal geometry affects the regulator's motion, and thus the inclination angle φ. Moreover, varying manufacturing conditions and process parameters lead to an additional variation of the shrinkage. Furthermore, due to the far lower Young's modulus of the thermoplastic (compared to the outer arm; steel), the inner arm has to be considered non-rigid (flexible). The resulting operation-dependent deformation of the inner arm also affects the inclination. In Figure 5, the appearing deviations are allocated to the stage of the product lifecycle in which they appear.

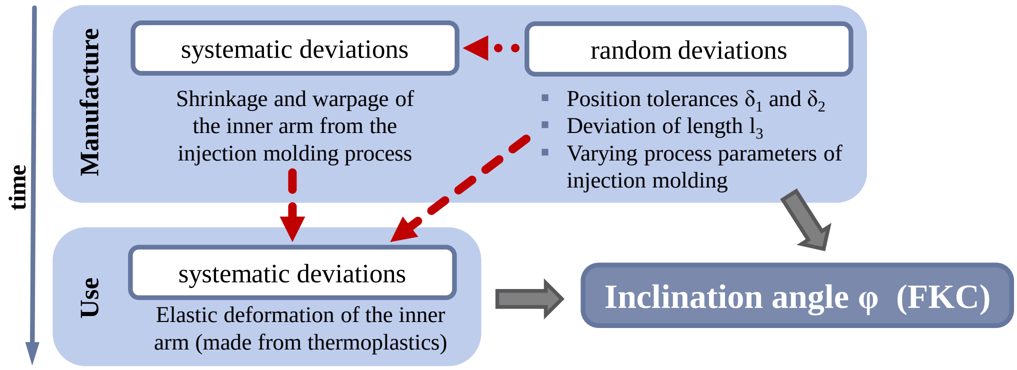



*Figure 5. Deviations of regulator and identification of appearing interactions (red arrows)*

As shown in Figure 5, the different kinds of deviations affect the inclination angle φ. However they also have effects among themselves – the so-called interactions (Walter et al., 2012):

- Interaction #1: The varying parameters of the injection molding process affect the systematic, manufacturing-caused shrinkage of the inner arm (red dotted arrow).
- Interaction #2: The random manufacturing deviations ($l_3$, $\delta_1$ and $\delta_2$) and the arm's shrinkage have effects on the operation-dependent deformation of the inner arm (red dashed arrows).

Consequently, two "interactions between deviations" of the regulator can be identified.

### 5.3 Step 3: Tolerance specification of random deviations

The statistical tolerance analysis of the regulator requires the specification of all tolerances and corresponding distributions. However, since the systematic deviations are deterministic, they must be established for each generated virtual mechanism of the regulator. Consequently, the tolerances are specified for the random manufacturing-caused deviations of the regulator's arms:

- Deviation in length $l_3 = 195.6 \pm 0.1$ mm (distribution: uniform)
- Position tolerance of joint axis $\delta_1 = 0.2$ mm (distribution: uniform)
- Position tolerance of joint axis $\delta_2 = 0.2$ mm (distribution: uniform)

The varying process parameters of the injection molding process are specified in section 5.6 (step 6).

### 5.4 Step 4: Determination of the functional relation (using vector-chains)

According to the methodology, a mathematical relation is needed that describes the dependencies between the FKC (inclination angle φ) and the appearing deviations of the mechanism. Therefore, several techniques are used in tolerance-related publications (such as vector-chains and T-Maps®) to establish the required functional relation (Stuppy and Meerkamm, 2009). According to Figure 4, the inclination angle φ of the window results from the height difference Δh between the window's points of support $P_1$ and $P_2$:

$$\varphi = \arctan\left(\frac{\Delta h}{L}\right) = \arctan\left(\frac{Y_{P_2}-Y_{P_1}}{X_{P_2}-X_{P_1}}\right) \tag{1}$$

Vector chains can be used to determine the current X- and Y-coordinates of the support points $P_1$ and $P_2$. These vector chains include i.e. the appearing deviation in $l_3$, position deviations $\delta_1$ and $\delta_2$ (with the corresponding excentricities $\varepsilon_i$ and angles $\vartheta_i$; representation of deviation in cylindrical polar coordinates) as well as the inner arm's deformation (with its two components $Def_X$ and $Def_Y$). Consequently, φ can be described as a function of the regulator's characteristics and deviations:

$$\varphi = f\,(l_1, l_2, l_3, l_4, \alpha, \varepsilon_1, \vartheta_1, \varepsilon_2, \vartheta_2, Def_X, Def_Y) \tag{2}$$

### 5.5 Step 5: Investigation of the effects on the FKC caused by interactions

Basically, with functional relation (2) a statistical tolerance analysis can be performed. However, the operation-dependent deviations $Def_X$ and $Def_Y$ first have to be determined. Since these are affected by the interactions, the effects of the interactions on the deformation should be investigated. If the interactions have no significant effects on $Def_X$ and $Def_Y$, the interactions may be neglected and thus the determinations in step 6 be simplified. Therefore, determination of the deformation for the regulator's worst-case tolerance specifications is needed. These results indicate on a significant impact of the interactions on the appearing deformations (as detailed in the final results in Figure 8).

### 5.6 Step 6: Determination of the deviations, which are affected by interactions

As mentioned before, $Def_X$ and $Def_Y$ must be determined for each of the virtual regulators (samples). However, the number of samples of a tolerance analysis is usually high, and thus computationally expensive. Consequently, a mathematical model is needed that represents the affected systematic deviations and the interactions towards them. Therefore, meta-models are used, which are integrated into the functional relation (2). According to Kleijnen (2009), a meta-model is "an approximation of the multi-input/multi-output relations given by the simulation model". The generation/training of a meta-model requires the determination of the affected deviation for a destined number of virtual mechanisms. This number is usually far less than the tolerance analysis requires (in this case 60).

Consequently, at first the appearing shrinkage and warpage of the inner arm must be determined for a destined number of samples (N = 60), using numerical injection molding simulations (e.g. Moldflow®). This simulation of the inner arm's shrinkage is conducted for each sample, with varying manufacturing-caused parameters (according to manufacturer's specification of the short fiber reinforced polymer PA66). The 60 varying molding parameter-sets were generated using Latin-Hypercube-Sampling. The appearing variations of the injection molding process parameters are:

- filling rate: 20 cm³/s ± 5 % (distribution: uniform)
- mold temperature: 85 °C ± 5 °C (distribution: uniform)
- melt temperature: 300 °C ± 10 °C (distribution: uniform)
- dwell/pack pressure: 40 % ± 2 °C (distribution: uniform)

- holding time of pack: 19 s ± 5 % (distribution: uniform)
- cooling time: 25 s ± 5 % (distribution: uniform)

Figure 6 shows the filling of the inner arm's injection mold at three points in time (filling time: 4 s).

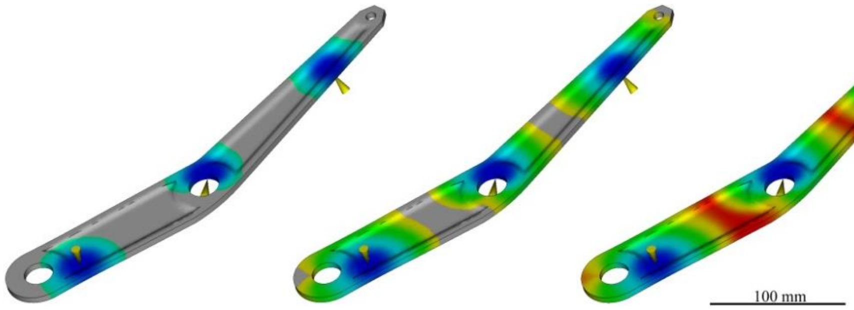



*Figure 6. Filling of mold (grey): (left) t = 1.5 s; (middle) t = 3.1 s; (right) finalized process*

The results of these simulations are 60 (slightly different) geometries (FE-meshes) of the inner arm, which represent the first interactions. Since varying shrinkage affects the kinematic behavior and thus the deformation (interaction #2), the 60 meshes are integrated into multi-body-dynamics simulations (MBD) with a flexible inner arm to determine the arm's deformation for each sample. The resulting deformations $Def_X$ and $Def_Y$ of the nominal regulator's inner arm are detailed in Figure 7 (right).

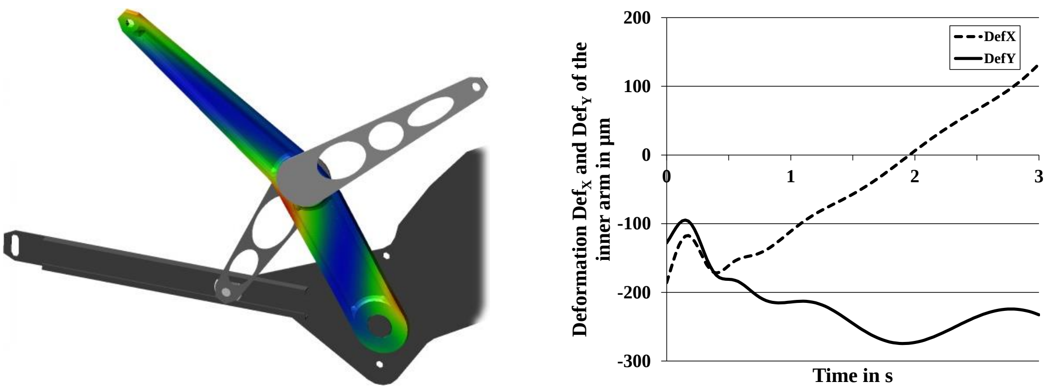



*Figure 7. (left) MBD of regulator (upper position); (right) $Def_X$ and $Def_Y$ of (nominal) arm*

### 5.7 Step 7: Generation/training of meta-models, representing the affected deviations

Based on the data-set determined (from the previous step), the required meta-models for each component of the inner arm's deformation $Def_X$ and $Def_Y$ can be generated/trained. These meta-models will replace the $Def_X$- and $Def_Y$-terms in the functional relation (2).

The data-set is divided into two sets – a so-called training set and the test set. Therefore, a Repeated Random Sub-Sampling with the commonly used ratio of 70:30 (training:testing) is used (Efron, 1982). The training set is used to train the meta-models – in this case: Artificial Neural Networks (ANNs).

An ANN is a mathematical model containing artificial neurons and connections among them:

$$y = f(\sum_i w_i \cdot x_i) \tag{7}$$

Each neuron responses with a certain output, depending on the synaptic weight $w_i$, the input-vector $x_i$ and the transfer function f (usually sigmoid functions are used for approximations). Hence, the trained ANN can calculate the corresponding output y to a combination of inputs $x_i$ (Andolfatto et al, 2012).

However, since meta-models are just approximations of the "real" dependencies, the prediction qualities of the two meta-models have to be evaluated. Therefore, a so-called "goodness-of-fit"parameter (like the mean squared error and the coefficient of prognosis COP) can be determined, using the remaining 18 samples (= 30 % of the entire data-set). According to Most and Will (2008) the COP ranges between 0 and 1, whereas a COP of 0.5 is equal to a prediction quality of 50 %.

The ANNs of $Def_X$ and $Def_Y$ are trained with 2 hidden layers (7 neurons on each layer). They achieve very high prediction qualities of $COP_{DefX} = 0.999$ and $COP_{DefY} = 0.998$. Consequently, the generated ANNs are reliable and thus can be integrated into the tolerance analysis of the window regulator.

### 5.8 Step 8: Choice and application of the tolerance analysis method

With the generation of the ANNs, all variables of the functional relation (2) are available and the inclination angle φ can be determined for any tolerance specification of the window's regulator. Subsequently, the FKC φ is determined for a very large number of virtual window regulators (samples), which just differ in the values of the appearing random deviations. This determination is done for 100,000 samples (using a Monte-Carlo-Simulation) to ensure statistical reliability.

### 5.9 Steps 9 and 10: Determination, representation and interpretation of the results

Finally, the results of the statistical tolerance analysis can be determined as well as visualized. Therefore, the final data-set of the previous steps (time-dependent inclination angles of each of the 100,000 virtual regulators) are used to determine the time-dependent variation of the FKC φ. An appropriate visualization of these results is shown in Figure 8 (left). This three-dimensional histogram-plot details the frequency distribution of the inclination angle φ during the entire motion sequence of the window regulator (time to lift the window: 3 s). Moreover, this visualization shows that the angle exceeds its given lower specification limit of $LSL_{\varphi}$ = -0.1 ° both at the beginning (t = 0 s) and during the last second of motion. The functional requirements of the window regulator cannot be ensured.

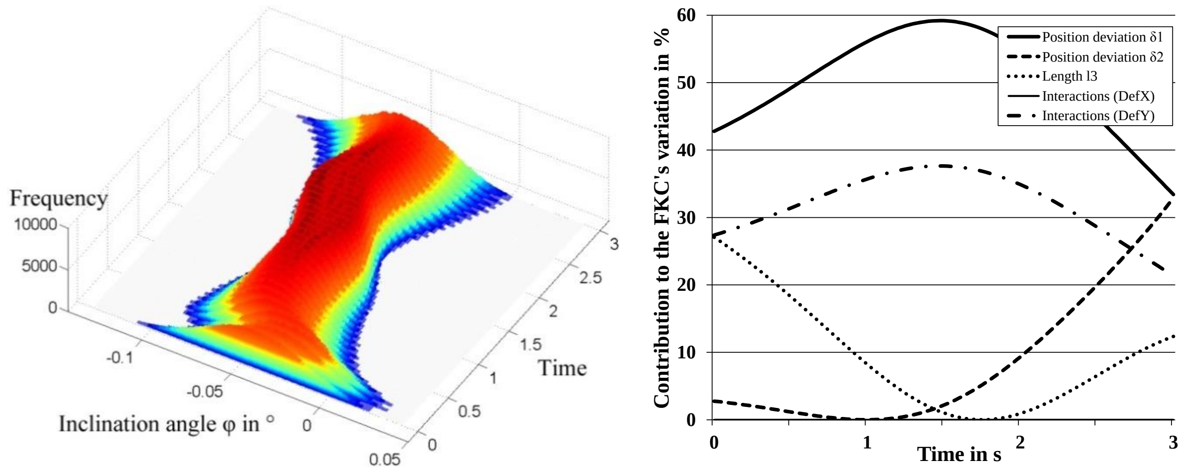



*Figure 8. (left) Time-dependent FKC φ; (right) Time-dependent contributor analysis*

Consequently, the product developer has to modify the tolerances (tolerance synthesis). Therefore, a second visualization of the time-dependent contributions of each deviation to the FKC's variation is shown in Figure 8 (right). Since the position tolerances $\delta_1$ and $\delta_2$ have a significant impact on φ, these should be narrowed. Moreover, the interactions cause/contribute more than 20 % of the FKC's variation (at t = 3 s). Consequently, their reduction would result in a helpful reduction in the FKC's variation. Finally, the reduction of the FKC's mean shift (caused by the systematic deformations) would also be recommended. This can be achieved by increasing the deformation $Def_Y$ (significant contributor) using e.g. short fiber-reinforced polymers instead of the non-reinforced thermoplastics.

## 6 CLOSING WORDS

This paper presented a methodology on statistical tolerance analyses of time-dependent mechanisms, which underlie different kinds of deviations. These deviations cause a variation in the system's FKC. Moreover, interactions between these deviations also appear which similarly affect the FKC.

Especially for less complex mechanisms, the functional relation can be quite easily set up by hand. Consequently, the tolerance analysis can be easily performed using e.g. Excel. Moreover, a large diversity of freeware and open-source work on meta-modeling techniques is available, which are easy to use stand-alone solutions (like RapidMiner®), or e.g. integrated codes like Matlab's neural network-toolbox. Hence, the product developer has appropriate support (methodology and tools) to set up a statistical tolerance analysis of a mechanism with deviations in 10 easy steps.

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