A GENERIC APPROACH TO SENSITIVITY ANALYSIS IN GEOMETRIC VARIATIONS MANAGEMENT

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Abstract
Engineering design teams are increasingly required to consider sustainability in the design of products and related manufacturing and assembly processes. In this regard, the economic and ecological sustainability of products as well as their quality and cost is influenced by geometric part deviations, which are inevitably observed on every manufactured artefact. Thus, the management of geometric deviations along the product lifecycle is a key issue for companies to living up to modern innovation requirements. The aim of this paper is to provide an overview of tools for the tolerance analysis in different stages of geometric variations management and to propose a versatile approach for the sensitivity analysis applicable in these different stages. This approach is to support decision making in the geometric variations management during integrated product and process design.

Keywords: Robust design, Tolerance representation and management, Decision making

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1 INTRODUCTION

Sustainability strongly gains importance in engineering design (Winkelmann, 2013) and adds to established innovation requirements, such as product quality, time to market, and product costs (ElMaraghy and ElMaraghy, 2014). Consequently, designers are increasingly required to consider product and process sustainability in the design of products, services, and systems. This is especially challenging, since engineering design can be seen as a decision process under uncertainty, where the uncertainties diversely influence the product properties (Dantan et al., 2013). Some of these uncertainties are geometric part deviations, which are inevitably observed on every manufactured artefact due to the axioms of manufacturing imprecision and measurement uncertainty (Srinivasan, 2006). It has been reported, that these geometric part deviations not only influence the product quality and cost (Wartzack et al., 2011), but also affect the economic and ecological sustainability (Hoffenson et al., 2013a, 2013b). Thus, there exists a strong need for the management of geometric part and product variations throughout the whole product lifecycle in order to ensure that the final product meets customer demands regarding not only product quality and cost but also sustainability.

From an engineering design perspective, geometric variations management can be seen as a branch of robust design, which covers all systematic efforts to reduce product sensitivity to noise factors (Arvidsson and Gremyr, 2008; Hasenkamp et al., 2009), since it deals with noise factors, which are related to part and product geometry (Schleich et al., 2014b). However, many companies struggle with the application of adequate methods and tools for early geometric variations management and often disregard the link between geometric product specification activities, product design, and manufacturing as well as assembly process planning. This can be traced back to a general lack of quantitative models for decision making support in robust design (Eifler et al., 2013), though the early design verification and validation is an important driver for industrial competitiveness (Maropoulos and Ceglarek, 2010). In particular, the lack of such quantitative models hinders the application of sensitivity analysis approaches, which support decision making and should hence be an essential part in the assessment of uncertainties (Plishchke et al., 2013) starting from conceptual design (Eifler et al., 2011), since it "provides insights [...] which input uncertainties dominate the output uncertainties, and how to appropriately invest resources to reduce uncertainty in analysis results" (Helton and Oberkampf, 2004).

Thus, the aim of this paper is to provide an overview of tools for the (quantitative) tolerance analysis in different stages of the geometric variations management process and to propose a versatile approach for the sensitivity analysis applicable in these different stages. This approach is to support decision making in the geometric variations management during integrated product and process design.

2 GEOMETRIC VARIATIONS MANAGEMENT

As it has been argued, geometric variations management covers all activities, which are related to controlling geometric deviations throughout the product lifecycle with the aim to ensure that the product lives up to customer demands. In this context, manifold activities and tasks during product design, manufacturing, assembly, inspection and quality control as can be seen from Figure 1 have to be performed by different actors employing specialized tools (Schleich et al., 2014b; Schleich and Wartzack, 2014b). Furthermore, successful geometric variations management requires the establishment of closed learning loops between these activities (Krogstie et al., 2013).

![Figure 1. Geometric Variations Management Stages along the Product Origination Process](image-url)
Consequently, many issues have to be clarified and several decisions have to be made during the various stages of geometric variations management. Some of these questions are (see Figure 2):

- How sensitive is the product structure to geometric part deviations and which product structure should consequently be chosen?
- How are parts to be designed in order to be manufacturable and less sensitive to varying operating conditions, such as thermal expansion and elastic deformations, regarding geometric deviations on single part level?
- Which operating windows are admissible from a geometric point of view when considering physical phenomena?
- How precisely should single parts be manufactured in order to ensure the geometric requirements of the product? Which tolerance specifications are necessary to control relevant part deviations and how tight should the corresponding tolerances be chosen?
- Which manufacturing process should be chosen in order to enable the manufacturing of required part specifications? How can manufacturing process parameters be optimized regarding the geometric precision of relevant part specifications?
- Which assembly processes are adequate to meet geometric product requirements considering irreducible part variations?
- Which part specifications should be measured and how should the geometric product requirements be checked?

Furthermore, for each of these issues, different product characteristics, i.e., design parameters, are relevant. For example, the product structure itself is under change in early stages of product design, part design parameters can be changed during embodiment and detail design, part tolerances are usually specified in detail design, the assembly process and the assembly sequence are subject to change during design and assembly planning, the manufacturing process is selected during design or manufacturing planning, manufacturing process parameters are optimized during manufacturing, and the admissible operating windows are checked during testing.

However, for each of the mentioned questions in geometric variations management, quantitative models are required to analyse the effects of uncertain parameters on the geometric requirements and to explore which parameters influence the product requirements. Therefore, computer-aided tolerance analysis is a key activity in geometric variations management, since it enables the prediction of the effects of geometric part deviations on geometric product requirements considering the assembly process without the need for physical prototypes. In this regard, it can be found, that the task setting for the tolerance analysis can be roughly divided into part tolerancing, i.e., "How much part variation is admissible given a specific product and assembly design to ensure the required product quality level?", and in assembly design and process tolerancing, i.e., "How should the parts be assembled given an irreducible amount of part variation in order to achieve the required quality level?" (see Figure 3). Beside this, various other sub-issues are to be considered in tolerance analysis, such as the specification of admissible operating windows ("Which operating conditions are admissible considering thermal expansion or elastic deformations of parts given a specific product and assembly..."
design to ensure the required product quality level?" (Armillotta and Semeraro, 2013), or process-oriented tolerancing ("Which fixture tolerances are required to ensure the geometric part requirements in multi-station machining processes?"") (Abellán-Nebot et al., 2013), which can be classified as part tolerancing problems. Thus, an overview of quantitative tolerance analysis approaches for geometric variations management is given in the following section.

Figure 3. Tolerance Analysis as a Key Tool in Geometric Variations Management

3 TOLERANCE ANALYSIS APPROACHES IN GEOMETRIC VARIATIONS MANAGEMENT

Tolerance analysis aims at predicting the effects of inevitable geometric part deviations on geometric product requirements (Chase and Parkinson, 1991) and is a key element for reducing overall cost (Prisco and Giorleo, 2002) and increasing product robustness (Lorin et al., 2010). Consequently, many researchers have devoted their efforts to the development of tolerance analysis approaches from one-dimensional tolerance chains to three-dimensional tolerance simulation tools over the past decades. Furthermore, a considerable number of review papers exploring the similarities and differences of these approaches is available (Chase and Parkinson, 1991; Nigam and Turner, 1995; Hong and Chang, 2002; Prisco and Giorleo, 2002; Shah et al., 2007; Bo et al., 2013).

In this regard, different tolerance analysis approaches for different stages of the product development process have been proposed. For example, tolerance analysis approaches from conceptual to detail design are presented by Johannesson and Söderberg (2000), which ground on function-means modelling for tolerance chain detection (Söderberg and Johannesson, 1999) during conceptual design, on part locating schemes in early embodiment design, and on the use of classical computer-aided tolerancing (CAT) tools in later embodiment and detail design. Moreover, an analysis tool for the robustness evaluation of rigid and compliant parts has been proposed by Söderberg et al. (2006b), which also employs function-means modelling and locating schemes for evaluating the degree of coupling and the stability analysis. Furthermore, approaches to support the design of locating systems is proposed by Söderberg et al. (2006a), which also grounds on methods similar to the aforementioned.

The issue of deriving tolerance specifications from product data in later embodiment and detail design stages is discussed by Armillotta (2013), where a software tool for the specification of geometric tolerances is highlighted. Beside this, with the aim to widen part tolerances and finally to reduce manufacturing costs, a tolerance synthesis and part optimization approach has been presented by Anselmetti (2010), which employs an optimization procedure to fulfil all functional requirements by adjusting part dimensions and their tolerances. Similarly to this, a software prototype for functional dimensioning and tolerancing is described by Islam (2004). An approach for the tolerance design based on skeleton models, which contain basic geometric information about the product, such as the product structure, the space claim of single components, and the interface locations, has been presented by Ziegler and Wartzack (2013). Methods for the tolerance analysis in early stages based on dimension chains are furthermore given in (Mannewitz, 2004). Beside these works on different tolerance analysis models, the issue of placing them in an integrated design workflow has been addressed in scientific literature. For example, an integrated design method employing assembly oriented graphs, propagation chains and risk analysis, as well as tolerance analysis was proposed by Mathieu and Marguet (2001). Furthermore, an integrated tolerancing process for conceptual design was introduced in (Dantan et al., 2003). With the aim to integrate the product modelling stage in tolerancing, the "geometry as soon as possible"-idea was proposed by Ballu et al.
(2006), which employs function-means modelling, assembly graphs, and tolerance analysis based on skeleton models.

Beside this, product models supporting the management of tolerancing information in collaborative design and process design have been investigated (Ballu et al., 2007; Dantan et al., 2007), also considering the traceability of geometric specifications (Jérome and Denis, 2008).

Based on the reviewed literature, it can be found, that approaches for the tolerance analysis applicable during different stages of product design are available. However, these approaches merely focus on typical design parameters, such as part dimensions and part tolerances. In contrast to this, part manufacturing, assembly process design, measurement planning, and the specification of admissible operating windows is to be considered in an integrated design process. Therefore, the approaches lack of a comprehensive and holistic view on the different issues in geometric variations management and hardly support decision making in the different phases of geometric variations management. Thus, an approach for the tolerance analysis and sensitivity analysis applicable during the various stages of geometric variations management is proposed in the following section.

4 A GENERIC APPROACH TO SENSITIVITY ANALYSIS IN GEOMETRIC VARIATIONS MANAGEMENT

The virtual evaluation and quantification of the effects of design, assembly, and manufacturing parameters on geometric product key characteristics requires a comprehensive mathematical model for the computer-aided tolerance analysis. Furthermore, versatile approaches for the sensitivity analysis are indispensable in order to be able to analyse the effects of input variation on the outputs considering different kinds of decision problems in geometric variations management. Thus, the concept of Skin Model Shapes as a new paradigm for computer-aided tolerancing and approaches for the sensitivity analysis, such as density-based sensitivity indices, are highlighted in the following, before a generic framework for the sensitivity analysis in geometric variations management is derived.

4.1 The Concept of Skin Model Shapes for Computer-Aided Tolerancing

The mathematical representation of geometric deviations, geometric tolerances, and geometric product requirements is a key issue in tolerancing research and has led to a considerable number of different approaches and computer-aided tolerancing theories. However, in order to overcome the shortcomings of most of these established models, such as the incomplete consideration of form deviations and the partial non-conformance to international standards for the geometric product specification and verification (GPS), the concept of Skin Model Shapes has been proposed recently (Schleich et al., 2014a; Anwer et al., 2014). The basic idea behind Skin Model Shapes is the operationalization of the Skin Model as a fundamental concept in modern GPS standards. In this regard, Skin Model Shapes can be found as specific virtual part representatives considering geometric deviations brought in by manufacturing, assembly, and use, where discrete geometry representation schemes, such as point clouds and surface meshes, are employed for their mathematical description. These discrete geometry representation schemes are available throughout the whole product origination process, from design, where they can be obtained by tessellation from the nominal CAD model, to manufacturing, where they result from manufacturing process simulations, and to inspection, where they are gathered from optical or tactile measurement systems. Therefore, the concept of Skin Model Shapes is capable of handling all different sources of geometric deviations and their effects on the part shape (Schleich and Wartzack, 2013a). The tolerance analysis based on Skin Model Shapes is performed by subsequently generating, assembling, and virtually using these part representatives employing simulation models (Schleich and Wartzack, 2013b, 2014a) (as can be seen from Figure 4). Hence, the different decision parameters in geometric variations management can be considered in the generation, assembly, and usage simulation of Skin Model Shapes.

![Figure 4. Tolerance Analysis Procedure based on Skin Model Shapes](image-url)
4.2 Density-Based Sensitivity Analysis

Sensitivity analysis (SA) aims at studying the relationship between the different sources of uncertainty in the model input and uncertainty in the model output (Saltelli et al., 2008). In this regard, various approaches to sensitivity analysis exist, where some sensitivity indices for the evaluation of solution variants in conceptual design have been reviewed in (Eifler et al., 2011). In particular, different sensitivity analysis approaches have been applied in the context of geometric variations management and tolerance analysis, such as the arithmetical contributor analysis, statistical contributor analysis and high-low-median sensitivity analysis (Stuppy and Meerkamm, 2009), which are local SA methods, as well as a global variance-based SA framework (Ziegler and Wartzack, 2015). However, current approaches to sensitivity analysis employ moment-independent sensitivity indices (Borgonovo, 2007), which overcome the limitations of variance-based SA methods and "unveil statistical dependencies that would not be captured using variance-based statistics" (Plischke et al., 2013). They ground on the estimation of conditional probability densities and are therefore called density-based sensitivity indices, where details can be found e.g. in (Borgonovo, 2007). In the following, an approach to estimating these global density-based sensitivity measures from given data as presented in (Plischke et al., 2013) is applied.

4.3 A Sensitivity Analysis Approach for Geometric Variations Management

As geometric variations management covers manifold issues and decisions, a generic framework for the sensitivity analysis is required, which should be applicable for the design synthesis (part and tolerance design), the manufacturing and assembly process selection, as well as the process optimization. In this regard, the coupling of quantitative tolerance analysis models and sensitivity analysis approaches is not without pitfalls, since e.g. parameter correlations may hinder the application of well-known variance-based sensitivity indices. Therefore, a sensitivity analysis framework is proposed, which is based on the tolerance analysis based on Skin Model Shapes and is thus capable of simulating the effects of different sources for part shape variations on geometric product requirements, and of various sensitivity analysis approaches, such as graphical ones in early stages of geometric variations management and density-based sensitivity indices in later stages, since these provide information about the importance of input variations on the geometric product requirements though correlations of input variation. The generic framework can be seen from Figure 5.
the tolerance analysis based on Skin Model Shapes and the connection of resulting input and output parameters by methods for the sensitivity analysis, such as graphical approaches in early stages of product design and density-based methods, e.g., presented by Plischke et al. (2013) in later stages. The straightforward application of the framework is highlighted in the following section.

5 APPLICATIONS

In the following, the sensitivity analysis framework for geometric variations management is illustrated in two case studies, where the first aims to illustrate the possibilities of early quantitative tolerance analysis coupled with straightforward graphical result interpretation for the product design, and the second highlights the application of the framework in integrated product and process design.

5.1 Product Design Optimization

The first study case is a stack-up of up to four parts as can be seen from Figure 6 (left). The geometric requirements of this stack-up are the height $h$ of the top plate and its parallelism $\text{para}$ to the ground. The designer is to choose between three different design configurations 1, 2, and 3 differing in the number and arrangement of parts, where it is known, that each of these four parts will have geometric deviations as can be seen from Figure 6 (right). In particular flatness deviations of the bottom and top mating plane ($\text{flt}$, $\text{flb}$), parallelism deviations between the top and the bottom mating plane ($\text{par}$), and position deviations between these planes ($\text{pos}$), which apply to all four parts.

![Figure 6. Tolerance Stack-Up for different Part Configurations and Part Tolerancing Scheme](image)

A tolerance analysis for the three configurations is performed by successively generating, assembling, and measuring Skin Model Shapes according to the tolerance specifications (see Figure 7, left). Thereafter, the probability densities of the product requirements are estimated from the 1,000 assemblies and are shown in Figure 7 (right). It can be seen, that configuration 3 leads to a wider range of the height (both minimal and maximal) as well as a stronger variation of the parallelism deviation. In contrast to this, the configurations 1 and 2 show little differences regarding the variation of the product requirements. Based on these results, the product developer is enabled to decide an appropriate configuration from a geometric variations management point of view.

![Figure 7. Tolerance Specifications and Probability Densities for the Product Requirements](image)
5.2 Injection Molding Process Optimization

The second study case is an adaption of the one presented in (Schleich and Wartzack, 2013a; Schleich et al., 2012) and consists of two flat plates manufactured by injection molding, which are assembled in an assembly fixture (see Figure 8). The requirements are the total length $d$ of the assembly measured from the assembly fixture and the parallelism $\text{para}$ of the back plane of the second plate to the fixture.

![Figure 8. Assembly of two Injection Molding Parts considering Shrinkage and Warpage](image)

In order to evaluate the effects of variation in the manufacturing process parameters on the geometric product requirements, a stochastic injection molding simulation is performed. Thereafter, the resulting geometries considering warpage and shrinkage are exported and used as Skin Model Shapes for the tolerance analysis. Finally, a density-based sensitivity analysis is performed, where the results are given in Figure 9. It can be found, that the main contributor to the variation of the assembly length is the variation of the melt temperature, whereas also the cooling time, the dwell time and the melt-flow-rate (MFR) affect the parallelism deviations of the assembly. Furthermore, the assembly tends to be more sensitive to variation of the first part, since the assembly orientation is influenced by the contact between the assembly fixture and the first part. Based on this information, part design can be adjusted and the injection molding process can be optimized.

![Figure 9. Results of the Sensitivity Analysis for the Injection Molding Case Study](image)

6 SUMMARY AND OUTLOOK

The management of geometric variations throughout the product life cycle is eminent not only from the quality and cost perspective but also considering product and process sustainability. However, it covers manifold issues and decisions, which have to be made during various stages of product and process design. Thus, with the aim to support decision making in integrated product and process design, a generic sensitivity analysis framework for geometric variations management was proposed. It grounds on the concept of Skin Model Shapes as a new paradigm for the computer aided tolerance analysis as well as graphical and density-based sensitivity analysis approaches. The usefulness of this framework was highlighted in two case studies covering the product design and manufacturing process optimization. However, future research will focus on the consideration of usage parameters in the tolerance analysis and the application of this framework to more complex decision making issues in geometric variations management.
REFERENCES


