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Sampling-based Tolerance-Cost Optimization of Systems with Interrelated Key Characteristics

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Abstract

Robustness is a key factor contributing to a high functionality of technical systems under uncertainty. In this context, existing methodologies, such as Axiomatic Design and Quality Function Deployment, can help to identify coupled functional requirements which significantly affect the robustness of a system. However, it is often not possible to decouple all in the final product design. As a consequence, multiple interrelated tolerance chains have to be considered in the subsequent tolerance design leading to multi-constrained or multi-objective optimization problems. Despite their significant influence on the optimization process and its results, interrelated tolerance chains have not been studied in detail yet, especially in the context of sampling-based tolerance-cost optimization. Moreover, a holistic framework for the tolerance-cost optimization of systems with multiple key characteristics is missing so far. In order to close that gap, this paper presents a framework to consider multiple key characteristics in both least-cost and best-quality tolerance-cost optimization using sampling techniques for tolerance analysis. Therefore, interrelated tolerance chains and their effects on the optimization process in terms of the definition and handling of multiple constraints and objectives are discussed in detail. The proposed research aims to bring all important aspects together in one common framework. Thus, it is supposed to help researchers and practitioners to properly define and solve the tolerance optimization problem. In order to show its benefits and applicability, it is applied to an illustrative case study. The novelty of this paper is to present a comprehensive method to support the tolerance engineer in creating a robust and optimal tolerance design of products with multiple interrelated key characteristics.

Keywords: Tolerance-cost optimization; Optimal tolerance allocation; Interrelated key characteristics; Multi-objective optimization

1. Introduction and Motivation

Geometric variations are inevitable [1] and thus have to be limited by tolerances to assure the functionality [2, 3] and the perceived quality of a product [4] over the entire product-life cycle [2]. However, tolerance allocation can become a complex and challenging task [5]: While loose tolerances enable cost-efficient manufacturing but lower the product quality, tight tolerances may lead to high-quality products but consequence increased costs [6, 7]. In order to solve this conflict, optimization-based tolerance allocation, i.e. tolerance-cost optimization, is well established [8]. Mainly due to its constantly increasing capability and performance, tolerance-cost optimization can nowadays be used to optimize complex products with numerous sub-assemblies and parts [8, 9].

However, increasing product complexity can lead to multiple, conflicting key characteristics [10, 11]. Despite the application of existing methods to initially decouple the key characteristics, interrelated tolerance chains cannot be totally avoided [11] and have to be adequately considered in the subsequent tolerance optimization process [8]. This implies a proper handling of multi-constrained and multi-objective optimization problems which have not fully been studied for sampling-based tolerance-cost optimization techniques yet. With the aim to overcome this drawback, the following article presents a novel framework which is used to help the user in the definition of both least-cost and best-quality tolerance-cost optimization of products with interrelated key characteristics. After discussing the current state of the art and related work in section 2, the proper consideration of interrelated tolerance chains in tolerance-cost optimization is discussed and integrated in one common framework in section 3. Its exemplary application in section 4 shows its applicability and benefits. Finally, a conclusion and an outlook are given in section 5.

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Nomenclature

$C_{(sum)}$	(Cumulated) Manufacturing costs
(F)KC,Y	(Functional) Key characteristic
g	Indicator function
I	Total number of tolerances
J	Total number of process alternatives
k	Single FKC
K	Total number of FKCs
LHS	Latin Hypercube Sampling
LSL,USL	Lower, upper specification limit
MO	Multi-objective
n	Sample size
SO	Single-objective
t	Tolerance
x_{ij}	Machine selection parameter
X	Dimension
z_{max}, $\hat{z}$	Maximum scrap rate, estimated scrap rate
ϱ	Probability density function

2. Literature review

Tolerance allocation plays an important role in the product development process to ensure the fulfillment of specified quality requirements which are influenced by unavoidable geometric part variations [1, 2]. Therefore, previous drawings, expert appraisals and experimental data mostly serve as a basis to select the tolerance values for the current design. The usage of manual, iterative approaches such as tolerance analysis methods in combination with sensitivity analysis, is common to check and assign the tolerance values on a trial-and-error basis [5]. However, these unsystematic approaches lead to non-optimal tolerance designs since the cost aspect is only indirectly considered by qualitative thumb rules [5, 12]. In contrast, tolerance-cost optimization overcomes this drawback by using optimization algorithms to solve the tolerance-cost problem taking both quantitative cost and quality information into account [8]. Since its beginnings in the mid 20th century, the method has continuously been enhanced over the last decades [8]. As a result, tolerance-cost optimization can successfully be used to:

- optimize complex, time-variant technical systems under uncertainties and environmental influences, e.g. forces or temperature [13, 14, 15],
- identify the global optimum for a wide range of tolerance-cost models [8, 16],
- select cost-efficient process and machine alternatives [6, 17],
- consider the quality loss for the customer [8, 15, 18],
- concurrently optimize design and manufacturing tolerances [8, 19] and
- simultaneously take both design parameters and tolerances into account [15, 20]

while statistically ensuring the fulfillment of the product requirements taking both dimensional and geometrical tolerances into account [8, 9, 21].

Therefore, the product requirements have to be broken down to assembly and part level and have to be described by the so-called key characteristics (KCs) [22]. Hence, a KC represents an important attribute which significantly influences a product requirement, such as cost, performance, functionality or safety, when the KC varies from its nominal [22]. In the context of tolerance design, geometrical KCs are mainly defined to ensure the functionality of a product and thus are also called functional key characteristics (FKC) [9, 10]. In order to comply with the product requirements, multiple KCs are required for even relatively simple products [22, 23]. Depending on their correlation, they are independent from each other or interrelated [8, 11, 24]. As it is exemplarily shown in Fig. 1, Y_1 and Y_2 or Y_2 and Y_3 are coupled by one common element X_{2b} or X_{3b}. Thus they are called interrelated or connected and can conflict [10, 23, 24]. In contrast, Y_1 and Y_3 are independent from each other.

In order to achieve a robust design, there are numerous approaches fostering the intended solution of the KC conflicts [25]. Depending on the degree of abstraction in the respective stages of the product development process, they support their early detection and, if possible, their decoupling. Thus, at the beginning of product development, Quality Function Deployment [26] and Axiomatic design [27] can be used to identify interactions between requirements leading to interrelated KCs in the subsequent development stages. Moreover, a comprehensive un- and decoupling can be achieved during the product design, e.g. by modifying the concept of a system [28], changing the assembly sequence [11], using fixtures [25] or applying tolerance compensation methods [29, 30].

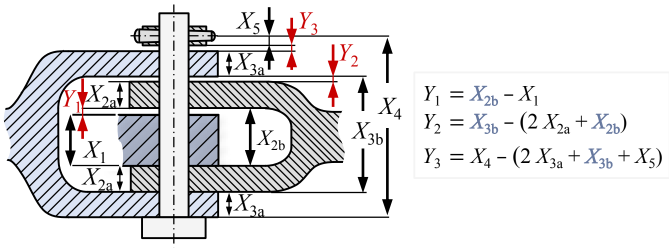

Fig. 1. Independent and interrelated KCs in comparison based on [17].

However, despite the rigorous application of these methods in the product development process, it is not always possible to avoid interrelated KCs in a final design [31]. Thus, they have consequently to be considered in tolerance-cost optimization. The effects of connected KCs in tolerance-cost optimization using worst-case or statistical tolerance analysis approaches, have been studied in detail yet, especially the proper formulation of multi-constrained optimization problems with the help of Lagrange-multipliers [8]. In contrast, they have not thoroughly been discussed in context of sampling-based tolerance-cost optimization, especially in best-quality tolerance-cost optimization. In addition, a framework to enhance the usability of both least-cost and best-quality tolerance-cost optimization is missing so far.

3. Tolerance-cost optimization of systems with interrelated key characteristics

After briefly discussing the fundamentals of tolerance-cost optimization in section 3.1, the effects of interrelated KCs are studied in section 3.2

3.1. Background

The global aim of tolerance-cost optimization is to identify an optimal combination of tolerance values $\boldsymbol{t} = [t_i, \ldots, t_I]^{\mathrm{T}}$ within their predefined limits $t_i \in [t_{i,\min}, t_{i,\max}]$. In general, tolerance-cost optimization can either be cost-driven or quality-driven to realize a least-cost or a best-quality tolerance design. The goal of least-cost tolerance optimization is to minimize the cumulated manufacturing costs C_{sum}(= objective) while meeting specified quality requirements (= constraint) [13]. In contrast, best-quality tolerance optimization tries to maximize the quality of a design e.g. by minimizing the scrap rate $\hat{z}$ (= objective) without exceeding a maximum cost limit $C_{\max}$ (= constraint). Both optimization problems are mathematically defined as it follows: [8, 13]

$$\begin{array}{lll|l} & \underline{\textit{Least-cost}} & & \underline{\textit{Best-quality}} \\ \text{Minimize} & C_{\mathrm{sum}}(\boldsymbol{t}), & & \hat{z}(\boldsymbol{t}), \\ \text{subject to :} & \hat{z}(\boldsymbol{t}) \leq z_{\max}. & & C_{\mathrm{sum}}(\boldsymbol{t}) \leq C_{\max}. \end{array} \tag{1}$$

Thus, the objective and the constraint are reversed depending on the chosen optimization goal (see Eq. (1)).

Initially, the potential of tolerance-cost optimization was limited by the existing computer technology and the restrictions of discrete optimization algorithms. Rising computing powers and the emergence of stochastic algorithms enhanced it to a powerful method solving a wide range of complex optimization problems (see section 2). Due to their problem-independent, user-friendly applicability, single- and multi-objective stochastic optimization algorithms, e.g. Simulated Annealing [19], Particle Swarm Optimization [10, 13], Genetic Algorithm (GA) [9, 21], Non-dominated Sorting Genetic Algorithm II (NSGA-II) [32] or Differential Evolution [32] are mainly used for tolerance-cost optimization today. The general workflow of tolerance-cost optimization is shown in Fig. 2. Starting with a set of initial tolerances, the optimization algorithm iteratively tries to adjust each tolerance t_i until a predefined termination criterion is met. Hence, the optimizer uses the information of previous iterations to select the current tolerances in each iteration for the subsequent tolerance and cost analysis. [9] Among worst-case and statistical tolerance analysis approaches, sampling techniques such as Monte Carlo Simulation or Latin Hypercube Sampling (LHS) are frequently used in sampling-based tolerance-cost optimization to analyze the variations of the KCs as a function of the varying inputs [3, 7, 9]. The resulting probability distribution serves as a basis to evaluate the effect of the currently selected tolerances on the functionality of the system by a suitable quality criterion, e.g. the scrap rate or non-conformity rate $\hat{z}_k$ predicted by appropriate estimation techniques [9, 33]:

$$\hat{z}(\boldsymbol{t}) = 1 - \int_{LSL}^{USL} \varrho(Y(\boldsymbol{t}))\, dx. \tag{2}$$

In addition, the costs for the current solution are analyzed using a tolerance-cost model to get the link between the currently allocated tolerances and the resulting manufacturing costs [9]. Therefore, experimental cost data serve as a basis to create the tolerance-cost relationships for each tolerance t_i [8] . The manufacturing costs C_{sum} for the total tolerance design are calculated as the sum of the individual manufacturing costs C_{ij} for a chosen process or machine alternative j [8]:

$$\begin{aligned} C_{\mathrm{sum}}(\boldsymbol{t}) &= \sum_{i=1}^{I}\sum_{j=1}^{J} x_{ij} \cdot C_{ij}, && \forall\ i = 1, \ldots, I, \\ x_{ij} &\in \{0; 1\}, && \forall\ j = 1, \ldots, J. \end{aligned} \tag{3}$$

This step is repeated until a balance between costs and quality is achieved and either a quality- or cost-optimal design is reached (see Fig. 2). However, Eq. (1-2) are limited to products with only one FKC. The effects of multiple FKCs on the optimization process with its constraint(s) and objective(s) are discussed in the following.

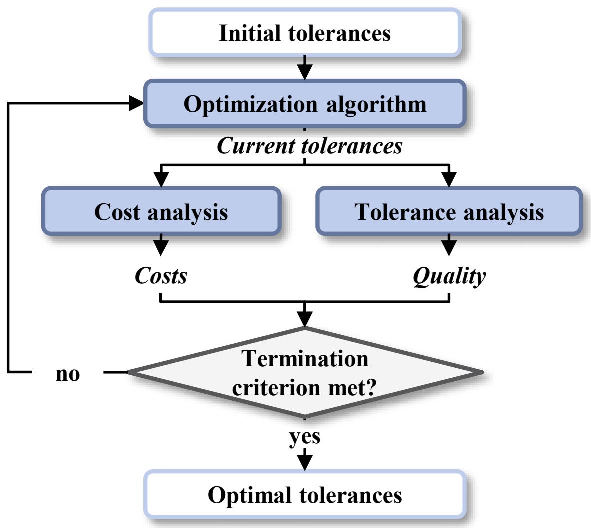

Fig. 2. Basic workflow of tolerance-cost optimization [9].

3.2. Interrelated FKCs in sampling-based tolerance-cost optimization

If the quality of a product is evaluated by measuring a single FKC, its total scrap rate $\hat{z}_{\mathrm{tot}}$ is equal to the scrap rate $\hat{z}_k$. When using sampling techniques in combination with an empirical scrap rate estimation technique $\hat{z}_{k=1}$ can be calculated as the ratio of the sum of defect parts to the total number n [9]:

$$\hat{z}_k = 1 - \frac{\sum_{i=1}^{n} g_k(Y_i)}{n}, \tag{4}$$

$$\textit{with}: \quad g_k(Y_i) = \begin{cases} 0 & \text{if} \quad Y_i < LSL_k, \\ 1 & \text{if} \quad LSL_k \le Y_i \le USL_k, \\ 0 & \text{if} \quad Y_i > USL_k. \end{cases} \tag{5}$$

For products with multiple, interrelated FKCs the functionality must be evaluated for each sample i and commonly considered by $\hat{z}_{\text{tot}}$:

$$\hat{z}_{\text{tot}} = 1 - \frac{\sum_{i=1}^{n} \prod_{k}^{K} g_k(Y_{i,k})}{n}. \tag{6}$$

This approach corresponds to a logical disjunction of the defects since the product is considered as non-functional when already one of all FKC specifications is unreached. Other approaches consequently lead to over- and underestimations of the product quality (see Fig. 3).

k \ i	1	2	3	4	5	…	n	Σ
Y_1	✓	✗	✓	✗	✓	…	✓	2
	∨	∨	∨	∨	∨	…	∨	
Y_2	✓	✗	✗	✓	✗	…	✗	4
	=	=	=	=	=	…	=	
Product	✓	✗	✗	✗	✗	…	✗	5

Fig. 3. Empirical scrap rate estimation considering multiple FKCs: ✗ = non-conform, ✓ = conform.

The number of defects of each individual FKC is lower than the product scrap rate since they are only separately considered. In addition, the calculation of $\hat{z}_k$ as the sum of the individually measured K scrap rates $\hat{z}_k$ leads to higher scrap rates (see Fig. 3). Consequently, the scrap rate estimation for multiple FKCs influences the definition and the results of both least-cost and best-quality tolerance-cost optimization.

However, before starting with the optimization, the designer should try his best to resolve any FKC conflict using the different methods highlighted in section 2 (see Fig. 4). After the final design is defined, the inevitable interrelated FKCs have to be considered in the tolerance-cost optimization. Thus, the optimization goal (least-cost or best-quality) has initially to be determined since it mostly affects the definition of the optimization problem (see Eq. (1)).

3.2.1. Least-cost tolerance-cost optimization

Aiming to achieve a least-cost design, the optimization problem is defined as a single-objective (SO) problem, regardless of the number of FKCs (see Eq. (1)). In contrast, multiple FKCs and their interrelation influence the handling of the constraint(s). If the tolerance analysis is based on worst-case or statistical approaches, e.g using the *root sum square* or the *estimated mean shift* method, the fulfillment of the functional requirements must be valued separately by K constraints [8]. Using sampling techniques, a decoupled quality assurance by defining K inequality conditions for each scrap rate $\hat{z}_k$ will lead to lower minimum manufacturing cost. However the maximum scrap rate for the whole product cannot be fulfilled (see section 4.3). Consequently, all K interrelated FKCs have to be commonly considered according to Eq. (6) which results in one single inequality constraint (see Fig. 4). Hence, the designer has to specify the maximum total scrap rate $z_{\max}$ and the maximum individual scrap rates $z_{k,\max}$. Any scrap rate $z_{k,\max}$ which is lower than the total scrap rate $z_{\max}$ has to be considered by an additional inequality constraint to meet the stricter quality requirements for the individual FKC (see Fig. 4).

3.2.2. Best-quality tolerance-cost optimization

In contrast, in best-quality tolerance-cost optimization the objective(s) to be minimized are the resulting scrap rate(s) as a function of the allocated tolerances (see Eq. (1)). By switching objective(s) and constraint(s), one constraint is sufficient to comply with the manufacturing cost limit $C_{\max}$ (see Fig. 1). With the aim to get an optimal overall scrap rate $\hat{z}_{\text{tot}}$, this problem can be solved by an single-objective optimization (see Fig. 4). However, the resulting scrap rates $\hat{z}_k$ cannot directly be controlled within the optimization process since they result from the identified tolerances. If the solutions are not to be restricted in advance, multi-objective (MO) optimization algorithms can be applied to simultaneously minimize each of the scrap rate $\hat{z}_k$ and $\hat{z}_{\text{tot}}$. The result is a Pareto front, a selection of non-dominated solutions [33, 35]. Its shape is influenced by the cost model and the correlation of the FKCs. This set of solutions can subsequently be used as a decision-making basis to prioritize the FKCs by choosing a suitable solution. [23, 33, 34] Hence, $\hat{z}_{\text{tot}}$ should always be defined as an objective since otherwise the total product quality cannot directly be optimized.

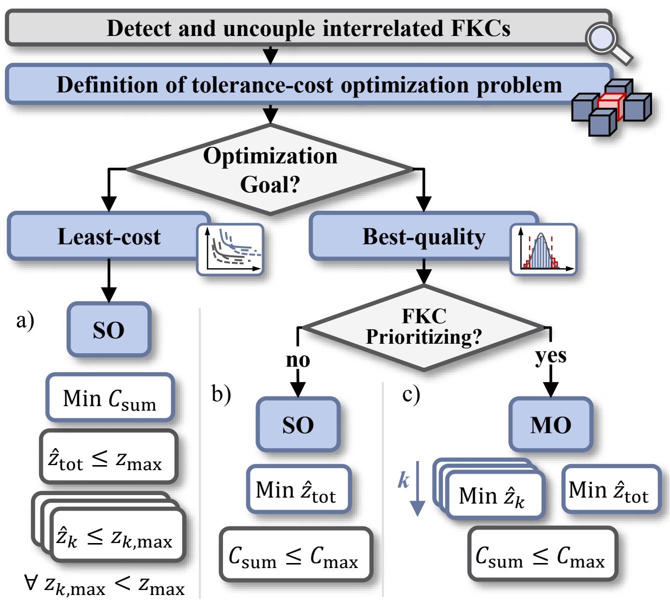

Fig. 4. Proposed framework to consider multiple FKCs in sampling-based tolerance-cost optimization.

4. Application

To verify the claims of section 3.2, both quality- and least-cost optimization are exemplarily applied to the knuckle joint illustrated in Fig. 1.

4.1. Presentation of the case study

Ensuring the functionality of the knuckle joint, three interrelated FKCs are taken into account (see Fig. 1). For least-cost optimization a maximum scrap rate $z_{\max} = z_{k,\max} = 2\,700$ ppm ($\hat{=} \pm 3\sigma$ [36]) must be fulfilled for the whole product and all FKCs. All tolerances are normally distributed with a standard deviation $s_i = t_i/6$. The cost model, process limits, nominal dimensions and specification limits are consistent to [17]. The maximum limit of the manufacturing costs for best-quality tolerance-optimization is set to $C_{\max} = 350$ \$.

4.2. Application of the proposed method

In order to ensure the comparability of the results, the same LHS is used for all studies and a sample size of $n = 100\,000$ was chosen as a compromise between computation time and quality of result (see [9]). The optimization is done in MATLAB®R2019a using a GA for single-objective and a NSGA-II for multi-objective optimization [35]. The population size was set to 200 (GA, NSGA-II), the total number of generations to 500 (GA) and 1000 (NSGA-II). An average relative change of 1e-6 in the best fitness function value over 250 generations was defined as termination criterion considering the scrap rates with an accuracy of one ppm for GA and NSGA-II. The default values were used for all other settings.

4.3. Discussion of the results

As it was claimed in section 3.2, the consideration of multiple FKCs influences the tolerance-cost optimization in terms of the resulting product quality and cost. Firstly, the effects of scrap rate estimation in **least-cost tolerance optimization** are discussed for the given case study. Thus, the product quality measured via the scrap rate(s) is considered by

a) one constraint: $\hat{z}_{\text{tot}} \leq z_{\max}$,

a*) K constraints: $\hat{z}_k \leq z_{k,\max}\ \forall\ k = 1, 2, 3$.

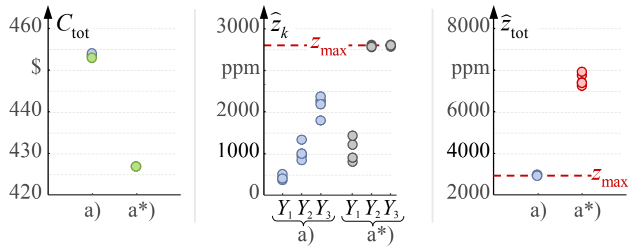

Fig. 5. Comparison of optimization results for least-cost optimization.

Fig. 5 compares the results for case a) and a*) for a five times repetition of the optimization based on the same LHS. Focusing on the objective, it can be seen that the decoupling of the FKCs for case a*) leads to lower minimum manufacturing costs than for case a) (see Fig. 5, left). The single scrap rates $\hat{z}_k$ for case a) are all coherently lower than for case a*) caused by the tighter tolerances (see Fig. 5, center). This further illustrates that Eq. 6 is a more restrictive criterion since it ensures total product quality for all FKCs. By an independent consideration of the FKCs, the total product quality cannot be guaranteed since it exceeds the maximum limit $z_{\max}$ (see Fig. 5, right). Therefore, the authors recommend case a) instead of case a*) to ensure overall product functionality (see Fig. 4).

Secondly, the effects of interrelated tolerance chains on the results of **best-quality tolerance optimization** are subsequently discussed. Depending on the number of objectives, the product quality can be optimized by: (see Fig.4, right)

b) one objective: $\hat{z}_{\text{tot}}$,

c) K+1 objectives: $\hat{z}_k, \hat{z}_{\text{tot}}\ \forall\ k = 1, 2, 3$.

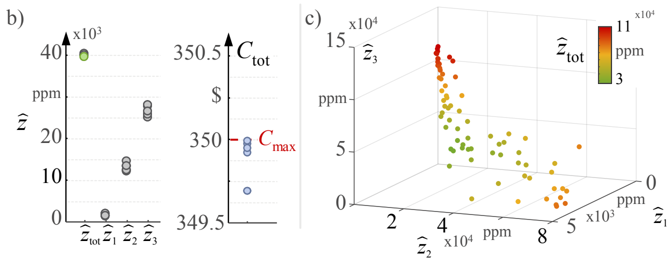

Fig. 6. Comparison of optimization results for best-quality optimization.

In accordance to Fig. 5, the optima for z_{tot} are considerably higher than the single scrap rates $\hat{z}_k$ (Fig. 6, left) for case b). In contrast to case b), $\hat{z}_{\text{tot}}$ and $\hat{z}_k$ are simultaneously optimized for case c) using the multi-objective optimizer NSGA-II to identify a set of optimal solutions. In Fig. 6 the best solutions are exemplarily shown for one optimization run. The estimated total scrap rate $\hat{z}_{\text{tot}}$ is visualized by the color of each marker. Depending on the number of the FKCs and their correlation, there are multiple solutions for comparable total scrap rates $\hat{z}_{\text{tot}}$ as a combination of the individual scrap rates $\hat{z}_k$. This set of solutions serves as a basis to select one configuration by prioritizing the objectives after the optimization. However, $\hat{z}_{\text{tot}}$ cannot be controlled directly if it is not defined as an objective.

5. Conclusion and Outlook

Although existing design methods can help to detect and decouple interrelated, conflicting key characteristics of complex products, they cannot be totally eliminated in the final design. Hence, this influences the subsequent tolerance-cost optimization process to create a balance between the resulting manufacturing costs and product quality. With the aim to support the user in properly defining and implementing sampling-based tolerance-cost optimization of products with interrelated key characteristics, all relevant aspects were combined in a common framework for both least-cost and best-quality tolerance-cost optimization and can serve as a guideline for both researchers and practitioners. Its exemplary application further emphasizes the importance of a proper application of scrap rate estimation in the tolerance-cost optimization process. Since the assembly sequence significantly influences the correlation of the different key characteristics, this potential should be further studied in the context of tolerance-cost optimization.

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