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Tolerance Analysis of Additively Manufactured Non-assembly Mechanisms considering Joint Clearance

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Abstract

Besides numerous advantages, Additive Manufacturing enables the manufacturing of entire non-assembly mechanisms within one single process step. However, this requires comparatively large joint clearances that significantly influence their functionality. Existing tolerance analysis methods completely neglect or simplify these clearances and thus cannot adequately represent a realistic motion behavior. With the aim to overcome this drawback, this contribution presents a novel method for the statistical tolerance analysis of additively manufactured mechanisms. The integration of results from multi-body simulation enables a realistic representation of the movements within the joints. Finally, the exemplary application to a planar non-assembly mechanism shows, that the process-related large joint clearances have more significant influence than the part deviations.

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1. Introduction and Motivation

Additive Manufacturing (AM) has recently attracted the attention of research and industry in the field of non-assembly products [1]. In contrast to conventional manufacturing processes, it offers the possibility of printing entire non-assembly mechanisms within one single process step, whereby a previously required assembly step becomes dispensable [2, 3]. However, additively manufactured joints require comparatively large clearances to ensure their separability. Minimum achievable joint clearances hereby depend on joint geometry, printer hardware and the chosen process parameters [4]. A trial and error process mostly optimizes these clearances, so they are tight enough to ensure the functionality, but large enough to avoid merging of mating surfaces [5, 6]. In addition, both small and large clearance can cause impairment and lack of accuracy of the motion behavior due to unsteady motion within the joints or vibration and instability. Motivated by the fact that large clearances of additively manufactured non-assembly mechanical joints play a crucial role in their kinematic behavior, their consideration in tolerance analysis is essential [7, 8].

Despite the advantage of manufacturing non-assembly mechanisms within a single process step, AM implies some shortcomings as for example process-related part deviations, which can affect certain part properties and thus impair their functionality [9]. In order to consider these deviations, a tolerance analysis for additively manufactured mechanisms in motion is essential. Statistical tolerance analysis methods are suitable for an early determination of the motion behavior of linkage mechanisms under uncertainty, but most of these methods imply drawbacks, such as the neglect and simplification of joint clearance. The state of the art concerning statistical tolerance analysis and analysis of mechanisms in general, currently insufficiently offers methods for systems in motion, let alone additively manufactured non-assembly systems in motion with consideration of large joint clearance [8]. The aim of this contribution is the consideration of joint clearance and AM-related deviations in the statistical tolerance analysis by the integration of results from multi-body simulation (MBS). Thus, a realistic representation of the movement within the joints is achieved.

The paper is structured as follows. The state of the art and related work will be reflected in the next section. Section 3 introduces the statistical tolerance analysis based on vector loops. Thereafter, a novel methodology for statistical tolerance analysis including consideration of joint clearance through results from MBS is applied to a planar, additively manufactured non-assembly mechanism in section 4. Finally, section 5 gives a brief conclusion and an outlook on further research activities.

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Nomenclature

c	Clearance
$\vec{c}$	Clearance vector
g	Number of linkages
J	Joint
L	Linkage
LHS	Latin-Hypercube-Sampling
MBS	Multibody Simulation
MCS	Monte-Carlo-Sampling
n	Number of joints
N	Sampling size
p	Number of vector loops
R_b	Radius of Bearing
R_j	Radius of Journal
t	Tolerance
γ	Force angle
θ	Angle
σ	Standard deviation
τ	Time

2. State of the art and related work

During the last decades, various approaches for the computer-aided tolerance analysis have been proposed. Three well known approaches are tolerance stacks, tolerance analysis based on the Small Displacement Torsor (SDT) and vector loops [10]. With the help of the vector loop approach, gaps between parts due to their geometric part deviations and kinematic variations can be considered [11]. Therefore, the assembly is modeled as a loop of vectors, each of them representing a dimension of the assembly. This can either be a dimensional part deviation, a geometric part deviation or a kinematic variation [10]. Methods of tolerance analysis including the consideration of joint clearance have mainly been developed in the field of robotics, whereby modifications in well-known models of kinematic analysis have been introduced [12]. Another approach is the enhancement of the vector loop approach, the clearance vector model. Therefore the clearance is modeled as a virtual, massless linkage connecting the two centre point of the joint pair respective to its clearance [13]. This clearance vector can be defined according to worst-case or stochastic scenarios or due to a certain joint force [14]. LANKRANI and NIKRAVESH proposed a force model for the modeling of joint clearance in which both elastic and damping effects are considered, whereby the damping effect is linked to the energy dissipated during the impact process. This model can be used in order to describe the dynamics of mechanisms including planar revolute clearance joints [15]. The modeling of joint clearance has been studied in detail in [16, 17]. For the purpose of analysing systems in motion, GARRETT and HALL presented a statistical approach to calculation of mechanical errors due to tolerances and joint clearance and introduced the term mobility band for the representation of these errors [18]. STUPPY ET AL. introduced the

“integrated tolerance analysis of systems in motion”, by which the tolerance analysis of a mechanisms with deviations through both manufacturing and operating can be conducted [14]. WALTER ET AL. extended this methodology by the consideration of interactions between deviations for systems in motion [19]. According to FLORES ET AL., there are three main modeling strategies for mechanisms in motion affected by joint clearance, the massless link approach, the spring-damper approach, and the momentum exchange approach [13]. Regarding modeling the impact of journal and bearing in MBS, two types of methods are firmly established, the continuous and the discontinuous approach. In the continuous contact model, the forces resulting from the collision act perpendicular to the plane of collision, whereby the model can be linear (Kelvin-Voigt model) or nonlinear (Hertz law) [13]. RHYU and KWAK presented an optimization approach for the design of mechanisms in which both tolerances and joint clearances were taken into account. This methodology was then applied to a planar four-bar mechanism including several joints affected by clearance [20]. Although tolerance analysis has been steadily improved over the past decades, most commercial programs present kinematic joints as ideal, neglecting joint clearance, friction and deformation within the joints [13, 21]. However, previous studies have shown that Additive Manufacturing of non-assembly mechanisms requires comparatively large joint clearances to ensure the separability of joint and bearing [22]. In order to ensure the functionality of the mechanism, the consideration of joint clearance in tolerance analysis is inevitable. To accomplish this, this paper presents a novel methodology for the statistical tolerance analysis of non-assembly systems in motion with consideration of joint clearance, whereby the required informations are derived through a MBS. Figure 1 shows an additively manufactured non-assembly joint in cross section with soluble support.

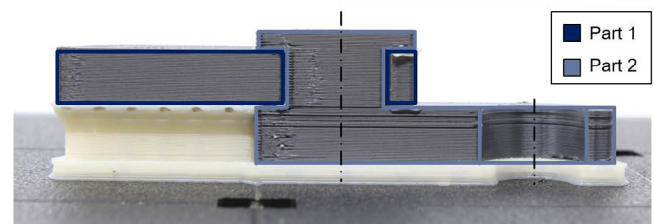


Fig. 1. Additively manufactured joint with soluble support

3. Tolerance Analysis considering joint clearance

Ensuring the functionality through tolerance analysis is a major step towards fully exploiting the potential of additively manufacturing non-assembly mechanisms. Therefore a methodology for a sampling-based statistical tolerance analysis considering both, tolerances and the AM process-related comparatively large joint clearances through results from a MBS is presented in this section. The first section describes the general approach of tolerance analysis for ideal mechanisms based

on vector loops. Subsequently, the method is enhanced by the clearance vector model through results from MBS, whereby the following tolerance analysis becomes more sophisticated. As a result of the presented methodology, a mobility band for the clearance affected moving system can be predicted, which illustrates the influence of deviations on the expected motion behavior of the joints [18]. Furthermore, the results of the mobility band serve as a basis for evaluating the influence of deviations and joint clearance on the movement behavior using a suitable sensitivity analysis technique. Thus the functionality of the mechanism can be ensured. The methodology is shown in Fig. 2.

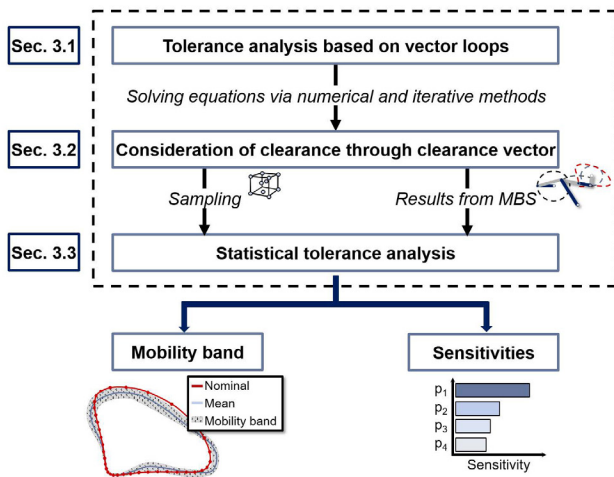


Fig. 2. Methodology for the tolerance analysis of systems in motion with consideration of joint clearance

3.1. Tolerance analysis of mechanisms based on vector loops neglecting joint clearance

This section describes the vector loop approach for statistical tolerance analysis for a mechanism in motion. Hereby the joints are assumed to be rigid and friction within the joints is neglected whereby the computing time can thus significantly be reduced. To show the applicability of this approach, it is applied to a planar 4-bar mechanism and each of the four links is represented by a vector (Fig. 3).

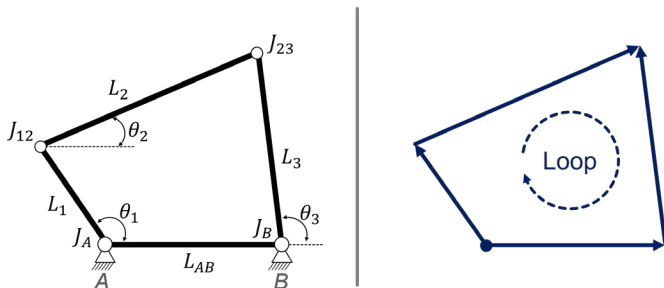


Fig. 3. Vector loop model of an ideal 4-bar-mechanism

For the vector loop approach, the loop closure equation according to Goessner is applied to determine the degrees of freedom and thus the number of required vector loops p for a mechanism including g linkages and n joints [23]:

$$p = g - (n - 1). \quad (1)$$

Applying this equation to the mechanism shown in Fig. 3, which includes four linkages and four joints, it becomes apparent that one vector loop is sufficient to characterize the kinematic behavior:

$$L_1 \cdot e^{j\theta_1} + L_2 \cdot e^{j\theta_2} - L_{AB} - L_3 \cdot e^{j\theta_3} = 0. \quad (2)$$

A vector loop as shown above consists of two equations for the real and the imaginary part, which both must be equal to zero:

$$L_i \cdot e^{\pm j\theta_i} = L_i \cdot (\cos\theta_i + j \cdot \sin\theta_i). \quad (3)$$

Thus for each vector loop, two equations are provided. For the 4-bar mechanism (see Fig. 3) one vector loop consisting of two equations is sufficient to describe its kinematic behavior. In the following only the summarized form of the equation, consisting of real and imaginary part will be shown. For this non-linear vector loop equations an explicit solution is difficult to solve, so a numerical solution is required. Therefore a numerical and iterative method (e.g. Newton-Raphson or Denavit-Hartenberg) is applied [11]. The movement accuracy for coupling curves of relevant joints J_{ij} as a function of the time τ is therefore defined as the functional key characteristic (FKC):

$$FKC(\tau) = J_{ij}(\tau). \quad (4)$$

The procedure for determining the unknown angles θ of the joints J_{ij} through solving the non-linear vector loop equations via numerical and iterative methods is shown in Fig. 4.

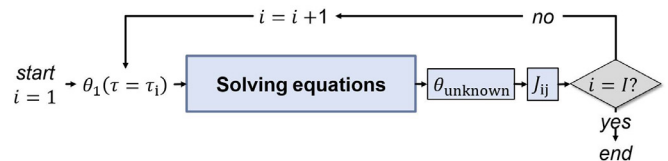


Fig. 4. Procedure for determining the unknown angles of the joints through solving the vector loop equations

3.2. Consideration of joint clearance through clearance vector model

The previously explained vector loop model is now enhanced through the clearance vector loop model to consider the influence of joint clearance on the kinematic behavior of systems in motion. For this, the clearance is modeled as the virtual, massless link connecting the two centre points of the joint pair, whereby the contact surface is assumed to be rigid and friction

is neglected. As friction within the joints is neglected, the direction of the clearance vector coincides with the normal direction of the collision plane. Under this assumption the clearance vector $\vec{c}$, consisting of the clearance c and the force angle γ , points in the same direction as the joint force [16, 17]. The modeling of the joint clearance and the resulting equivalent clearance link for the clearance vector model is shown in Fig. 5.

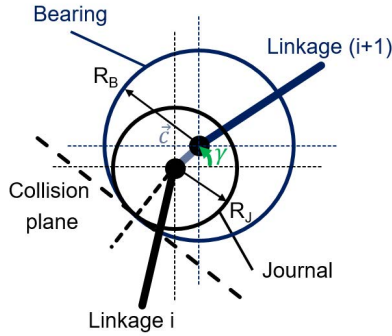


Fig. 5. Equivalent clearance link for clearance vector model

The joint forces and thus the clearance vectors for moving systems can be derived using common software programs, e.g. MSC ADAMS. With the help of the MBS, the missing informations for the clearance vector like the force angle γ can be determined for a certain modeling type. Subsequently, this information can be used for solving the clearance vector loop equations and then be integrated into the tolerance analysis for a realistic representation of the joint clearance. Furthermore, lubricants in the non-assembly mechanism are not existing, so a contact load can occur, whereby the resulting impulse forces are transferred to following mechanical parts. These impulses and the subsequent continuous contact can be modeled by the force model [8]. According to Flores et al. [13], the size of the clearance is hereby defined by the difference in radius between bearing R_b and journal R_j :

$$c = R_b - R_j. \quad (5)$$

The vector loop approach for the 4-bar mechanism of Fig. 3, is now extended by the clearance vectors $\vec{c}_{12}$ and $\vec{c}_{23}$ for the clearance affected joints J_{12} and J_{23} in Fig 6.

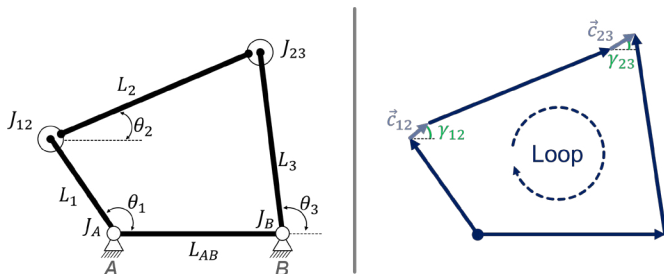


Fig. 6. Clearance vector loop model of a 4-bar-mechanism with joint clearance

According to Goessner (see Eq. 1), for a 4-bar mechanism consisting of four links g and four joints n , one vector loop

is sufficient for the characterization of its motion behavior, whereby the informations for the clearance vectors are derived through the MBS [23]:

$$L_1 \cdot e^{j\theta_1} + c_{12} \cdot e^{j\gamma_{12}} + L_2 \cdot e^{j\theta_2} + c_{23} \cdot e^{j\gamma_{23}} - L_{AB} - L_3 \cdot e^{j\theta_3} = 0. \quad (6)$$

3.3. Statistical tolerance analysis using sampling techniques

Statistical tolerance analysis using sampling techniques offers the possibility to determine the influence of joint clearances and geometrical deviations resulting from the AM-process on the movement behavior. Therefore, the link length deviations are considered in terms of tolerances whereby these values have to be determined through prior experiments, as there is currently no specific guideline for tolerancing in AM and in particular for non-assembly mechanisms [24]. Various techniques such as Monte-Carlo-Sampling (MCS) or Latin-Hypercube-Sampling (LHS) can be used for sampling. The adaption of the procedure for solving the vector closure equations from Fig 4 for solving the equations for the statistical tolerance analysis using sampling techniques is shown in Fig. 7.

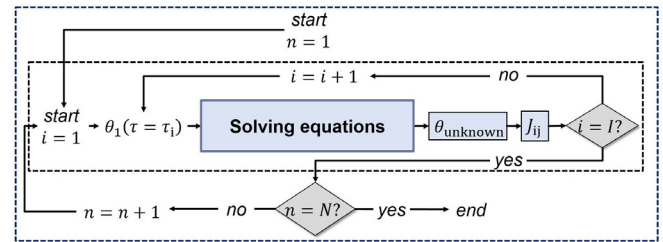


Fig. 7. Procedure for the statistical tolerance analysis using sampling technique

As a result, the mobility band for the 4-bar mechanism can be calculated whereby the functionality can be ensured, evaluating the previously defined FKC .

4. Case Study

The previously presented methodology is now applied to a case study of an additively non-assembly manufactured mechanism, consisting of eight linkages and seven clearance affected joints (see Fig. 8). By applying equation (1) to the case study, it becomes evident that two vector loops are now required for solving the vector loop model:

$$L_1 \cdot e^{j\theta_1} + c_{12} \cdot e^{j\gamma_{12}} + L_{2a} \cdot e^{j\theta_2} + c_{23} \cdot e^{j\gamma_{23}} - L_{AB} \cdot e^{j\theta_A} - L_3 \cdot e^{j\theta_3} = 0, \quad (7)$$

$$L_1 \cdot e^{j\theta_1} + c_{12} \cdot e^{j\gamma_{12}} + e^{j\theta_2} (L_{2a} + L_{2b} \cdot e^{-j\theta_{2a2b}}) + c_{23} \cdot e^{j\gamma_{23}} + c_{24} \cdot e^{j\gamma_{24}} + L_4 \cdot e^{j\theta_4} + c_{45} \cdot e^{j\gamma_{45}} + L_5 \cdot e^{j\theta_5} - L_{AC} \cdot e^{j\theta} = 0. \quad (8)$$

In order to determine the clearance vector for the enhancement of the vector loop model, as described previously, a MBS

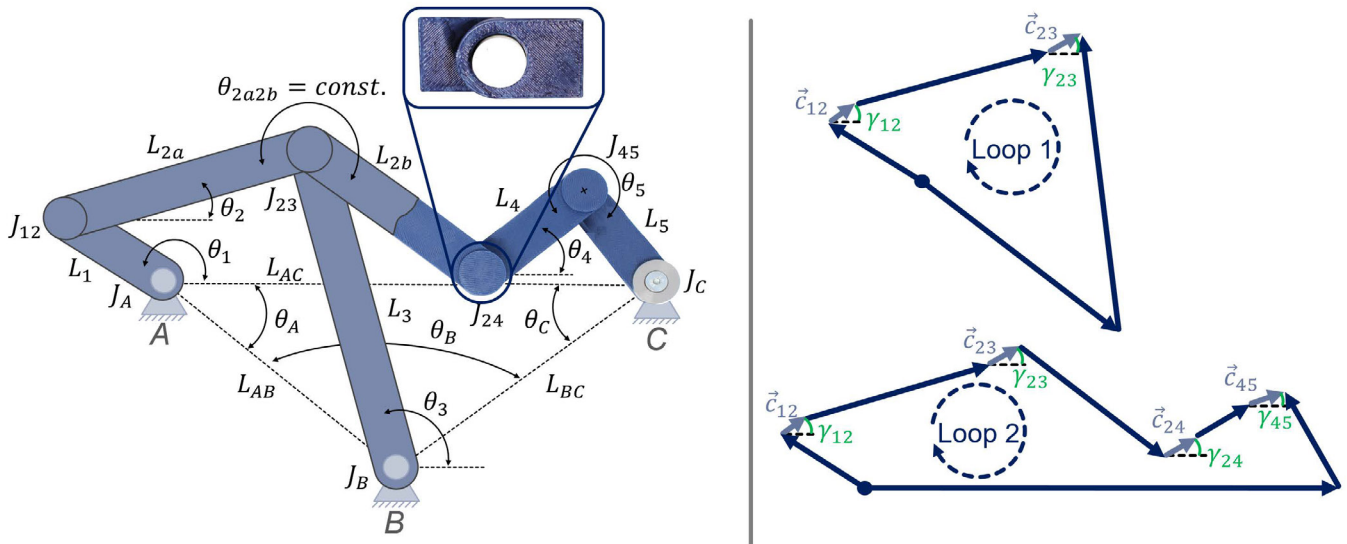


Fig. 8. Clearance vector loop model of an additively manufactured, non-assembly 8-bar mechanism

of the presented mechanism was setup in MSC ADAMS. Therefore the joints were modeled as rigid and affected by clearance. This clearance was varied between minimum achievable clearance and a comparatively large clearance and split into four factor levels for a comparison of the results (0.2mm, 0.5mm, 0.7mm and 1mm). This minimum achievable joint clearance was determined in prior studies whereby joints with different joint clearances and varying process parameters were printed. A minimum achievable joint clearance of 0.2mm was determined, whereby the separability between journal and bearing could still be guaranteed. Afterwards, the derived joint forces from the MBS were implemented into the tolerance analysis. In this case study, the motion behavior and in particular the movement accuracy of coupling curve of joint J_{24} is defined as the FKC as a function of time τ :

$$FKC(\tau) = L_1 \cdot e^{j\theta_1} + c_{12} \cdot e^{j\gamma_{12}} + e^{j\theta_2}(L_{2a} + L_{2b} \cdot e^{-i\theta_{2a2b}}) + c_{23} \cdot e^{j\gamma_{23}} \quad (9)$$

The deviations from the desired motion behavior are used for the sensitivity analysis in order to evaluate the influences of joint clearance and part deviation of the linkages on the defined FKC , whereby the results are filtered to avoid statistical outliers. The density-based sensitivity analysis is performed in MATLAB R2019a according to PLISCHKE [25] due to its sampling-independent applicability. The tolerance for the length of the linkages was fixed at a constant value ($t = \pm 0.2$), based on prior experiments in which the length deviation of the linkages resulting from the FDM process was examined in a design of experiment, printing different linkages with varying process parameters. The values of the tolerances were assumed to be normally distributed ($\sigma = t/6$). Furthermore the number of samples is $N = 10000$ with which valid results could be achieved in prior studies in combination with reasonable computing time [19]. The aim of this sensitivity analysis is to

identify which of these parameters have the most significant influence on the motion behavior of the additively manufactured non-assembly mechanism. This information can then be used to optimize the functionality of the mechanism more efficiently [26]. The resulting contributors of the sensitivity analysis are shown in a bar chart in Fig. 9.

It can be stated that the joint clearance has the most significant influence on the motion accuracy of the joint J_{24} . Regarding the sensitivities of the linkage deviation, the deviation of linkage L_3 has the greatest impact on the motion accuracy of the observed joint. This seems plausible, since it is the longest linkage in the assembly (cf. leverage effect).

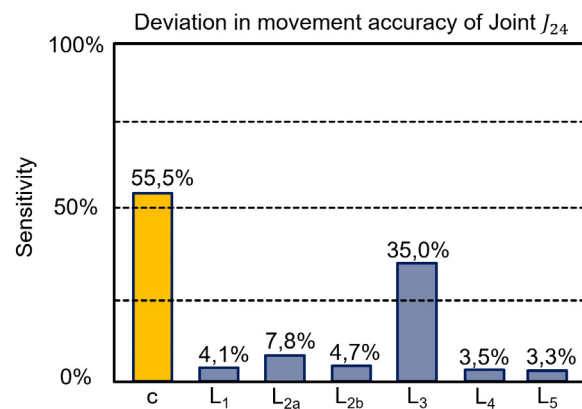


Fig. 9. Resulting sensitivities of tolerances and joint clearance on the motion accuracy of joint J_{24} for the given case study

5. Conclusion and Outlook

AM-specific, comparatively large joint clearances of additively manufactured non-assembly mechanisms have a signif-

icant influence on their motion behavior and can thus impair their functionality. By considering these large joint clearances through a clearance vector, whereby the required informations are derived through a MBS in statistical tolerance analysis, a realistic representation of the motion behavior can be achieved. In doing so, the functionality of the mechanism can be predicted. With the help of the presented statistical tolerance analysis, the effects of joint clearance and tolerances on the motion behavior of an additively manufactured non-assembly mechanism are calculated. Evaluating the sensitivity analysis in Fig. 9, it becomes evident that the joint clearance has a far more significant influence on the motion behavior of the mechanism than the tolerance of the linkage length. Furthermore, it can be stated that minimizing the joint clearance would significantly improve the movement accuracy of the joints and thus ensure the functionality.

For further improvement of the presented methodology, machine-specific deviations resulting from the AM-process (e.g. staircase effect, surface roughness) have to be considered for a realistic representation of the motion behavior. In order to additionally achieve a process-oriented prediction of machine-specific deviations depending on chosen production parameters, an integration of a suitable meta-model may be purposeful. Furthermore, the AM-specific form deviations could be considered through Skin Model Shapes [27], whereby the motion behavior of additively manufactured non-assembly mechanisms can be predicted more precisely. Another subject of future work will be the experimental validation of assumptions made, such as the neglect of 3D-effects like e.g. tilting and the continuous contact model, whereby free-flight phases and friction within the joints are neglected. The comparison of test and tolerance simulation can be achieved by using optical measurement techniques. An ubiquitous research topic concerning AM, is the optimization of process parameters, whereby the joint clearance and thus the motion behavior of non-assembly mechanisms can be significantly improved, whereby the potential of AM can be fully exploited.

Acknowledgements

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