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## CAD-BASED TOLERANCE ANALYSIS IN PRELIMINARY DESIGN STAGES ENABLING EARLY TOLERANCE EVALUATION

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### ABSTRACT

Associated with manufacturing and assembly processes, inevitable geometric deviations have a decisive influence on the function and quality of products. Therefore, their consideration and management are important tasks in product development. Moreover, to meet the demand for short development times, the front-loading of design processes is indispensable. This requires early tolerance analyses evaluating the effect of deviations in a design stage, where the product's geometry has not yet been finally defined.

Since such an early tolerance consideration allows quick and economic design changes seeking for robust designs, it is advisable that the design engineer, who is entirely familiar with the design, should take this step. For this purpose, this paper presents an easy-to-use CAD-based tolerance analysis method for skeleton models. The relevant part deviations are represented by varying geometric dimensions with externally driven family tables. The approach comprises the strength of vector-based methods but does not require an expensive set-up of tolerance analysis models. Particularly, the novelty of this method lies in the CAD-internal sampling-based tolerance analysis of simple geometries without the use of expensive CAT software. This enables designers to evaluate the effect of tolerances already at the preliminary design stage. Using a case study, the presented approach is compared with the conventional vector-based tolerance analysis.

**Keywords:** CAD-based tolerance analysis, early tolerance management, skeleton model

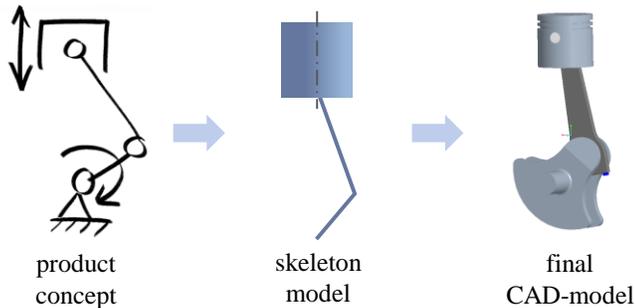
### NOMENCLATURE

ASME	American Society of Mechanical Engineers
CAD	Computer-Aided Design
CAT	Computer-Aided Tolerancing
DRF	Datum Reference Frame
FKC	Functional Key Characteristic
ISO	International Organization for Standardization
LHS	Latin Hypercube Sampling

### INTRODUCTION

Due to the multitude of determining parameters, the development process of technical products is usually characterized by many iterations. In order to reduce the number of expensive iterations, the consistent application of modern simulation software and process models, such as the design methodology from PAHL and BEITZ [1], is essential and widespread. According to the first-time-right principle, this enables an early prediction of the effects of design decisions. However, despite the considerable effect of geometric deviations, tolerances are usually considered in the end of the product development process, when the product's geometry is finally defined [2]. Taking into account the functional requirements, this often forces designers to assign tight tolerances, especially for parts of complex assemblies. In order to avoid excessive costs, time-consuming optimizations of a given tolerance design are commonly carried out in a subsequent step. However, since manufacturing costs are largely determined by decisions taken early in the product development process [3], cost-optimal solutions can rarely be found in this stage. Therefore, it would be useful that tolerances are considered during the whole design process and especially already in early design stages. Since the tolerance consideration in early design

stages is closely linked to the design process, it is advisable that the consideration is done by the designer themselves. Therefore, the skeleton model, which is an important part of a top-down-driven development of complex assemblies [4], is a proper starting point for a first quantitative tolerance analysis, see Figure 1. In the preliminary design stage, the simple two or three-dimensional structure including basic geometry elements [5] enables a first assignment of tolerances for skeletons [6, 7].



**Figure 1 Different geometric degrees of details in product development, exemplary for a thrust crank drive**

The tolerance analysis of fully defined CAD models, which is the scope of most commercial CAT software, is often difficult due to scarce resources. In combination with the low level of experience, their application is often a major obstacle for common designers. In order to partially overcome the problem, this paper proposes an approach that enables the designer to perform a simplified statistical tolerance analysis of skeleton models within his CAD system.

The paper is structured as follows. Initially, related work considering tolerance analysis particularly in CAD systems is presented. After a brief introduction of the vector-based tolerance analysis of skeleton models, the process of the proposed CAD-based tolerance analysis is thoroughly described. Subsequently, both methods are exemplarily applied to a scissors lift table. Finally, a conclusion and an outlook is given.

## STATE OF THE ART: TOLERANCE ANALYSIS

Due to the significant role in the product development process, the tolerance analysis methods are an important area of research [8]. Depending on the different application areas and the variety of potential users [9], there are numerous representation models used for tolerance analysis. DANTAN and QURESHI divide these analysis methods into two groups according to the type of accumulation [10].

The tolerance accumulation approach aims to represent the resulting multidimensional tolerance zone of the key characteristic by a combination of single tolerance zones based on the assigned tolerances [10]. The different forms of representation, such as T-Map<sup>®</sup> [11, 12], deviation domain [13] or specification hull [14], use the concept of Degree of Freedom describing the permissible deviation [15]. Although these methods are efficient for tolerance analysis [10], their application may be challenging for engineers [16, 17].

In contrast, deviation accumulation approaches describe the behavior of a key characteristic with a functional expression taking into account single deviations [10]. Among others, the accumulation of deviations can be described with matrix transforms [18], small displacement torsors [19], Jacobian matrices or vector loops [20]. By means of the formula expression, these methods help understanding the functional interrelationships. However, without the application of CAT software tools, such as RD&T<sup>®</sup>, 3DCS<sup>®</sup>, VSA<sup>®</sup>, Enventive<sup>®</sup> and CETOL6 $\sigma$ <sup>®</sup>, a parameterized system description according to the tolerance specifications can be difficult and time-consuming in the design process [17].

Since designers ask for easy-to-use tolerance analysis systems, one potential solution is the use of CAD models as the database for deriving mathematical models [21, 22]. However a CAD-based tolerance analysis model requires compatibility with standards, such as ASME Y14.5 or ISO 1101, as well as computability [23]. Since tolerance information in CAD systems is often merely used as annotation [24–26], the computer-processable description of geometric tolerances is challenging. Therefore, usually only linear, one-dimensional tolerance chains can be evaluated in common CAD systems [24, 27]. Motivated by the lack of a generally applicable CAD-integrated tolerance analysis [9], a vector-based analysis in CAD-systems is suggested [27]. This vectorial dimensioning and tolerancing is unambiguous [21] and similar with the Boundary Representation of the CAD model [27]. For example, GEIS et al. proposed an approach that uses surface attribute containers describing position and orientation deviations of surfaces with two vectors [27]. Likewise alternative approaches requiring an upstream definition of reference points or local coordinate systems in order to enable a vectorial description [6, 21] this method is predominantly applicable for tolerance engineers but not for designers. Another widely recognized problem is the proper definition of contact surfaces in assemblies. In case of over-constrained systems a redefinition of mating conditions is required [28, 29].

In summary, it can be concluded that the numerous available tolerance analysis methods are either complex or, as in the case of a one-dimensional CAD-based tolerance analysis, have little significance and are not universally applicable. Although the need and benefit for a tolerance analysis supporting the designer in the product development process is indisputable, methods for an early tolerance evaluation are lacking. This applies in particular to skeleton models that require a time-efficient statistical tolerance analysis.

## VECTOR-BASED TOLERANCE ANALYSIS OF SKELETON MODELS

As already mentioned, the vector-based approach is generally well suited for the analysis of skeleton models. In particular, the restriction to basic geometry elements allows an easily traceable vector description of the geometry and the tolerances. For instance, the linear dimension of a line of the skeleton is analogously represented by a single vector and its associated length. By arranging these vectors in chains and

loops, complete assembly skeleton models can be reproduced. In doing so, dimensional and geometric deviations as well as the kinematic conditions of adjacent components respectively vectors can be described [30]. Based on the procedure introduced by POLINI [20], the vector-based tolerance analysis of skeleton models is described below.

Starting from a skeleton model, the available information about dimensions, mating conditions and functional requirements helps to identify the necessary vector chains. Considering assemblies, a local datum reference frame (DRF) is then defined for each individual part or the respective representing elements in the skeleton. In a subsequent step, kinematic joints between the local DRFs are set according to the mating conditions such as the coincidence of points. In combination with the information from the skeleton model, open or closed vector loops will be created. This structure of vectors, representing individual skeleton elements, is mathematically described as a sequence of several rigid body transformation matrices [20]:

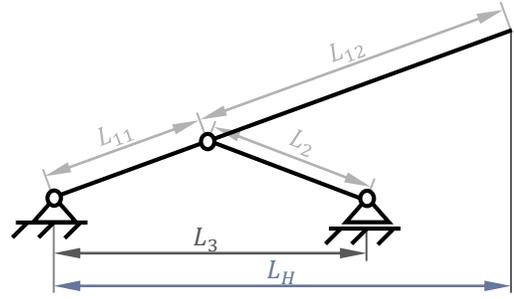
$$\mathbf{R}_1 * \mathbf{T}_1 * \dots * \mathbf{R}_i * \mathbf{T}_i * \dots * \mathbf{R}_n * \mathbf{T}_n * \mathbf{R}_f = \mathbf{H} \quad (1)$$

The rotational transformation matrices  $\mathbf{R}_i$  depict the rotation of the local datum reference frames of the individual vectors represented with translational matrices  $\mathbf{T}_i$ . The final rotational matrix  $\mathbf{R}_f$  rotates the matrix back into the initial coordinate system. The output matrix  $\mathbf{H}$  is equal to the identity matrix for closed loops or represents a resulting transformation matrix characterizing a functional characteristic. In the two-dimensional case the corresponding input matrices may have the following form [20]:

$$\mathbf{R}_i = \begin{bmatrix} \cos\alpha_i & \sin\alpha_i & 0 \\ \sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{T}_i = \begin{bmatrix} 1 & 0 & L_i \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2)$$

where  $\alpha_i$  is the angle between the individual vectors with the corresponding length  $L_i$ . For a subsequent tolerance analysis, the vector description of the components of the skeleton model is modified taking into account dimensional and geometric tolerances. In the simple case of a dimensional tolerance, this merely requires the variation of the length of the respective vectors. Considering the plain example of a 2D skeleton model shown in Figure 1, the vertical position of the piston can easily be analyzed with a singular open vector tolerance chain. This indicates that the vector-based approach is well suited for the analysis of simple structures.

However, its application can lead to a comprehensive and complex mathematical description of the tolerance analysis problem. Considering the example in Figure 2, the analysis of the functional key characteristic (FKC)  $L_H$  requires the setup of two vector loops.



**Figure 2 Example of a 2D skeleton model with dimensional tolerances**

First of all, the closed loop containing the vectors with the dimension  $L_{11}$ ,  $L_2$  and  $L_3$  indicates the tilt of the long strut. Together with a second open loop ( $L_{11}$ ,  $L_{12}$ ,  $L_H$ ), the following simplified mathematical description of the FKC results:

$$L_H = \frac{(L_{11} + L_{12}) * \left( L_3 - \frac{L_2^2 - L_{11}^2 + L_3^2}{2 * L_3} \right)}{L_{11}} \quad (3)$$

Although the resulting equation is relatively clear and comprehensible, its derivation using the vector approach is complex and time-consuming. This applies in particular to structures with many elements, as is common in industrial skeleton models. In addition to the consideration of geometric tolerances and mating conditions, the expansion to the three-dimensional space further heightens complexity. This implies that the error-prone process of the vector-based approach is only partially suitable for a quick tolerance analysis in preliminary design stages.

### CAD-BASED TOLERANCE ANALYSIS OF SKELETON MODELS

Since a parametric CAD skeleton model with relevant geometry and constraint information is often available anyway (e.g. for initial kinematic simulations), it is a logical step to perform the tolerance analysis directly in the CAD systems. Furthermore, the great similarity between CAD models and vector models [30], indicates the benefit of a CAD-based tolerance analysis.

The basic idea of the proposed approach is a CAD-internal tolerance analysis by controlling the dimensions of the skeleton model with the help of family tables. Contrary to the standard integrated tolerance tools, this allows a statistical tolerance analysis considering the effect of interdependencies. The approach is structured as shown in Fig. 3 and the corresponding steps are described in detail below.

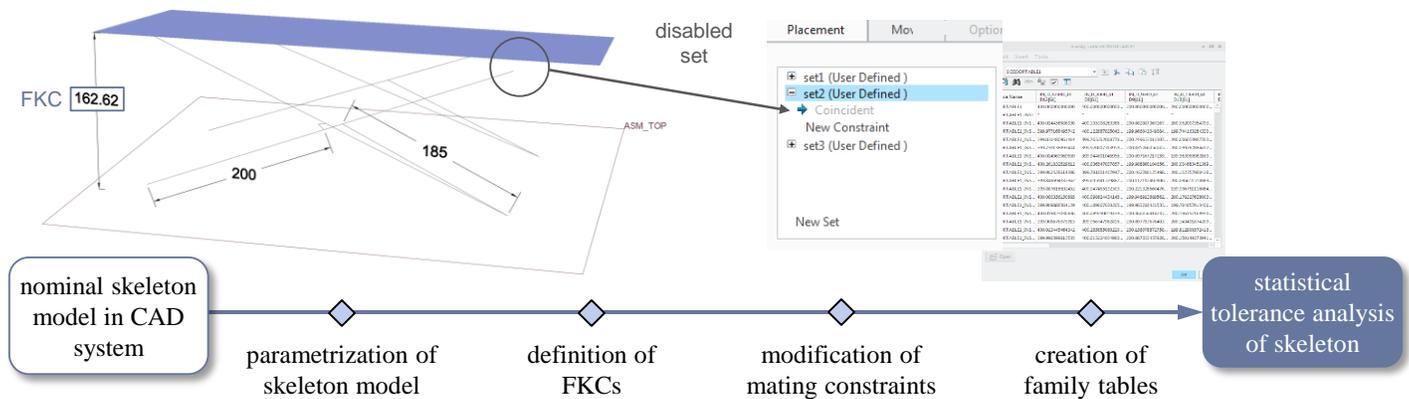


Figure 3 Process of the CAD-based tolerance analysis of skeleton models

### Step 1: Generation of nominal skeleton models

Although the creation of new skeleton models is part of the routine work, the designer should take in account some aspects that facilitate the consistent tolerance analysis. Considering the complexity of assemblies, it is useful to assemble the skeleton model from individual component skeletons according to the parts or sub-assemblies. This economizes the design process of skeleton models and allows a realistic assignment of mating constraints for adjacent components.

Furthermore, the constraints (partly unintended) defined in the sketch of skeletons play a crucial role, since they restrain changes of dimensions that are taken into account in the tolerance analysis. Therefore, it is good practice to remove some constraints and replace them with appropriate dimensions in order to control the deviating geometry. Considering two perpendicular lines, the perpendicularity constraint is deleted and an additional dimension of  $90^\circ$  is added enabling the depiction of angular deviations. Although this means a slight additional effort in the preliminary design stage, it helps the designer to understand how individual deviations affect the product.

### Step 2: Parametrization of the skeleton model

This comprehension of the effect of tolerances is further enhanced by the parametrization of the skeleton model in order to provide a variational model. Similar to the vector-based approach, the mapping of dimensional tolerances is trivial and in modern CAD systems such as Creo Parametric® it does not require a manual parametrization. In contrast, the description of geometric tolerances with mostly multidimensional tolerance zones (see Table 1) can be challenging [17]. For this reason, different approaches exist that allow an automatic generation of tolerance zones on the basis of assigned geometric tolerance annotations of CAD models [31]. However, as the tolerance information in preliminary design stages is usually abstract due to missing geometric details, the description of tolerance zones in accordance with standards often requires additional dimensions similar to the vector approach. In this context, additional axes, points and local reference coordinate systems

are defined in the CAD-based tolerance analysis of skeleton models. These newly added elements enable the mapping of multidimensional tolerance zones for skeleton models with simple geometry elements (see Table 1).

Table 1 Exemplary representation of geometric tolerances

tolerated element	tolerance	tolerance zone
point		2D 3D
2D line		2D
axis		3D

As shown in Figure 4, these additional elements lead to small topological changes of the skeleton model according to the effect of applied deviations. Thus, the nominal contact point between the struts 1 and 2 is replaced by the real contact point, in which the two contact points of the struts with position deviation coincide. This point lies in the intersection of the two tolerance zones and varies depending on the dimensions that describe these zones.

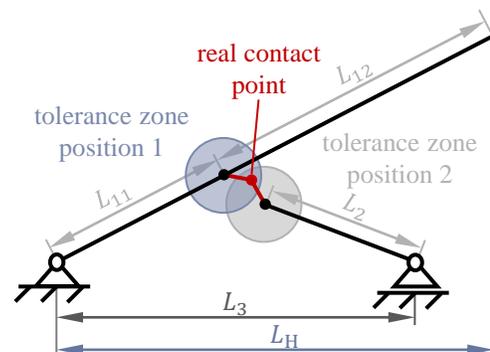
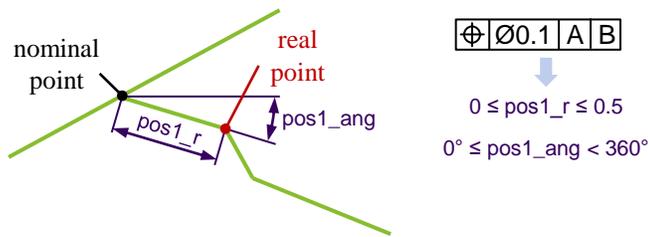


Figure 4 Minor topological changes of the skeleton by adding geometric tolerances

In case of the 2D position tolerance, shown in Figure 5, the corresponding tolerance zone is described with two independent dimensions. Since the definition of this independent tolerance parameters is advantageous for the subsequent tolerance analysis, it is advisable to select a suitable coordinate system according to the shape of the tolerance zone. For the example in Figure 5, this means that the circular zone is best described in a cylindrical coordinate system located in the nominal point. Corresponding to the assigned position tolerance, the real point must be within a circle with a radius ( $pos1\_r$ ) of 0.5 mm.

Since skeleton models of assemblies usually have little geometry elements, the effort for parameterization is limited to few relevant tolerances. This effort can be further reduced to a minimum if the influencing tolerances are already taken into account during the creation of new skeleton models.



**Figure 5 Description of two dimensional position tolerance**

### Step 3: Definition of Functional Key Characteristics

In a subsequent step, the FKC's are defined according to the list of requirements. In the skeleton model, the dimensional FKC's are easily evaluated using the measurement and analysis tools available in CAD systems. Contrary to the vector-based approach, this enables the simultaneous evaluation of several FKC's without significant effort.

### Step 4: Modification of mating constraints

Although small geometrical and dimensional deviations have only minor effects on the topology, the mating constraints between the individual component skeleton models have to be partially redefined. This applies in particular to over-constrained assemblies and mechanism with rigid parts. The mating constraints that change depending on the deviations can be respected in analogy to the vector-based tolerance analysis with a distinction of cases. Therefore, current CAD systems allow a conditional application of mating constraints by defining of relations. However, since robust designs are rarely over-constrained, a redefinition of constraints is usually not needed. After this step, a fully parameterized tolerance analysis model is available in the CAD system.

### Step 5: Creation of family tables

In addition to a worst-case analysis, the parametrized model also enables a statistical tolerance analysis and thus a reliable evaluation of the effect of individual deviations. In order to realize a sampling-based analysis, the predefined dimensions describing the deviations as well as the FKC's are compiled in a family table, see Table 2.

**Table 2 Structure of a family table**

name	pos1_r	pos1_ang	...	FKC LH
skeleton_instance1	0.15	127	...	60.53
skeleton_instance2	0.38	290	...	59.89
...	...	...	...	...

Therein, each sample point corresponds to an instance of the skeleton model containing an individual set of input parameters. According to the previously defined tolerances and the expected probability distribution, the values of each parameter are generated with an external sampling method. These values, generated in Excel® or MATLAB® for example, are finally transferred to the family tables of CAD systems.

The required number of samples necessary for reliable results strongly depends on the complexity of the system, the number of analyzed parameters and the sampling strategy. In case of the example shown in Figure 2 with 6 input parameters, a sample number of 1000 (Latin Hypercube Sampling - LHS) leads to reliable results. Due to the low complexity of the example, an increased number of samples only slightly improves the quality of the results. For example, the standard deviation of the resulting FKC  $L_H$  changes by only 1.3% with one million sample points, which is sufficiently accurate in this stage.

### Step 6: Statistical tolerance analysis of skeleton model

One option for performing the statistical tolerance analysis is to manually open, regenerate and evaluate the individual instances of the family table. Due to the fact that the modified mating constraints lead to topological changes depending on the combination of input parameters, a multiple regeneration of the model is necessary for a correct representation. However, since this procedure is time-consuming and conventional CAD systems allow an internal or external control of certain actions, this process is completely automated within the proposed approach. In the CAD system Creo Parametric 4.0®, which is used as an example, so-called trail files, storing certain actions for a particular working session, are used. A subsequent start of the program in batch mode allows a complete regeneration of all instances in the background enabling the parallel run of several tasks.

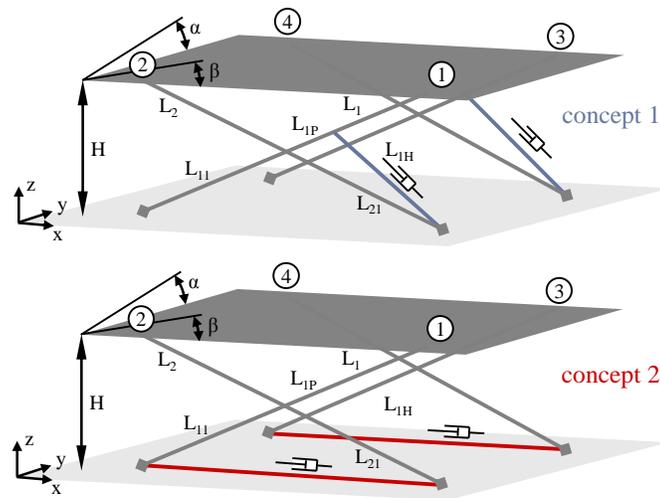
The resulting family table, which contains the values for the input parameters as well as for the FKC parameters, is used for the analysis with regard to the fulfillment of requirements. The probability distribution of the FKC thus shows the range of values and enables a first estimation of the expected scrap rates solely based on the skeleton model. This allows a selective narrowing or widening of individual tolerances while taking functionality and costs into account. In this step, scatter plots and correlation analyses of input and output parameters help to evaluate the effect of each deviation on the FKC. Moreover, sensitivity analysis methods can be used to quantify this effect. These measures allow an early functional validation of assemblies based on skeleton models. In order to obtain robust products, the early tolerance analysis in preliminary design stages enables changes of the tolerancing scheme as well as a major modification of the geometry with little effort.

## CASE STUDY

In order to show the usability and benefit of the process described above, the CAD-based tolerance analysis is exemplarily applied. Since the preliminary design stages are characterized by major design changes, the analysis is subsequently described in the context of a process for developing a robust product. Finally the results are compared with those of the conventional vector-based approach.

### Presentation of the case study

In this section, a hydraulic scissor lift table, as shown in Figure 6, is analyzed. This practical example demonstrates the challenges arising during the analysis of skeleton models of complex assemblies and allows the consideration of different concepts. However, the simple design and the clear FKCs (height  $H$  and tilt of the table  $\alpha$ ,  $\beta$ ) enhances traceability of the results.



**Figure 6** Skeleton model of a hydraulic scissor lift table for concept 1 (Figure 6a) and concept 2 (Figure 6b)

In concrete terms, two concepts are analyzed that differ significantly in terms of their robustness with regard to the FKCs. At first, the skeleton models are analyzed taking into account only dimensional tolerances in accordance with the general tolerances from Table 3. Finally, additional geometric deviations are taken into account for a more detailed analysis. For the subsequent statistical tolerance analysis, the tolerances are assumed to be normal distributed with a standard deviation of  $\pm 3\sigma$ .

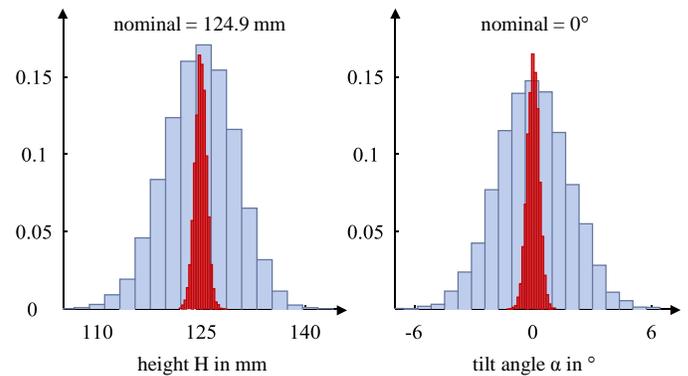
**Table 3** Parameterized dimensions with specified tolerances

dimension	name	nominal in mm	tolerance value
length of table leg	$L_1, L_2, L_3, L_4$	400	$\pm 0.5$
distance foot – contact point	$L_{11}, L_{21}, L_{31}, L_{41}$	200	$\pm 0.5$
distance foot – contact cylinder	$L_{1P}, L_{3P}$	220	$\pm 0.5$
length of hydr. cyl. concept 1	$L_{1H}, L_{3H}$	184.28	$\pm 0.5$
length of hydr. cyl. concept 2	$L_{1H}, L_{3H}$	380	$\pm 0.5$

### CAD-based tolerance analysis

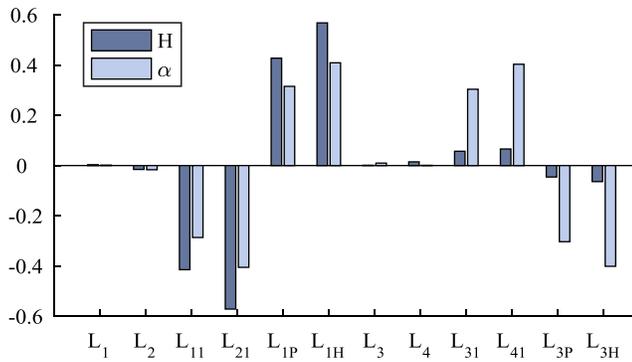
Since the skeleton model does not contain geometric details, some elements such as the mating constraints have an abstract character. According to the real mating constraints, the table plate is firmly connected to the legs in point 2 and point 4 by means of a pivot joint whereas the points 1 and 3 serve as support point, see Figure 6. The resulting over-constrained system requires a redefinition of mating conditions in the CAD system. This applies in particular to the support point, which changes depending on the z-coordinate of the points 1 and 3. Thus, the model is basically suitable for the CAD-based tolerance analysis.

Since the effort involved in creating an analysis model is low, analyzing the system solely based on predefined dimensions is a reasonable first step to estimate the behavior of the FKCs. Due to the long computing times (about 50 sample points per minute), the number of samples is set to 1000 (LHS) at this early stage for an initial qualitative estimation. The automated computation of the instances of the family table according to this sampling set shows the resulting values for the FKCs. This indicates that the height  $H$  and the tilt around the x-axis  $\alpha$  are subject to significant deviations for concept 1. The associated distributions of the results are depicted in the blue histograms in Figure 7.



**Figure 7** Histogram for height (left) and tilt angle (right). Results of concept 1 (blue) and concept 2 (red) are showed.

Since the variation of the resulting FKCs is not acceptable, the initial objective is to find out the main source for this system behavior. For this purpose, considering scatter plots and the correlation between deviating input parameters and FKCs enables the identification of the main contributing parameters. The Pearson correlation coefficients depicted in Figure 8 indicate that the distance between the foot and the contact point of the legs ( $L_{i1}$ ) and especially the position ( $L_{iP}$ ) and length of the hydraulic cylinder ( $L_{iH}$ ) have great influence on the height  $H$  and tilt angle  $\alpha$  of concept 1. Since the linear correlation coefficients for  $L_{1H}$  and  $L_{3H}$  have different algebraic signs a plain “synchronizing” of the length of both hydraulic cylinders, already leads to a reduced variation of the resulting tilt angle  $\alpha$ . In the real assembly, this would be equivalent to replacing the two hydraulic cylinders with one central cylinder.



**Figure 8 Pearson correlation of parameterized dimensions and the resulting key characteristics of concept 1**

However, since above all the height is still sensitive to variations in the cylinder length, the designer may initiate a redesign of the position of the hydraulic cylinder. Such conceptual changes can be easily implemented in the early preliminary design stages, because only a few adaptations of the design have to be made. For example, placing the hydraulic cylinders between the base of both scissors leads to the alternative concept 2 shown in Figure 6b. Still considering only predefined dimensions with the general tolerances from Table 3 the standard deviation of the height  $H$  and tilt angle  $\alpha$  of concept 2 is reduced to about one fifth of the original value, while the second tilt angle  $\beta$  is still negligible.

Since concept 2 seems to be robust, geometric deviations are taken into account in addition to dimensional deviations to further validate the FKCs. In concrete terms, position tolerances with a circular tolerance zone with a diameter of 1 mm are assigned to the crossing points of each legs. Since geometric reference elements are missing in the skeleton model, the designer is supposed to set the planes or axes of the coordinate system as reference. In addition, a virtual tilting of the legs from the  $x$ - $z$ -plane ( $\pm 0.3^\circ$ ) allows an abstract representation of angular deviations of the support at the base of the legs. Due to the additional consideration of geometric deviations, the number of varying input parameters increases to 16 (4 x length of legs, 8 x position of contact point, 2 x tilt of legs and 2 x length of cylinder). In order to obtain quantitative reliable results, the sampling number is correspondingly raised to 5000.

Compared to the analysis of concept 2 solely based on dimensional variations, the additional consideration of geometric deviations only leads to marginal changes in the probability distribution of the FKCs. The red histograms in Figure 7 show the variation of the resulting FKCs and clearly demonstrate the high robustness of concept 2. Therefore, the redesign of concepts is a proper way to improve the robustness of the product and should be done prior to a detailed design of tolerances.

## Results

For the analyzed model, the modification of mating constraints choosing the correct contact point has little influence on the resulting distributions and can be neglected in this case. However, since this statement is not universally applicable, a

redefinition of mating constraints is recommended in case of any doubts. The decision on whether the mating constraints need to be redefined also depends on the expected knowledge gain obtained by the analysis.

Accordingly, the CAD-based tolerance analysis can have different levels of detail. In early stages of preliminary design, in which the concept has not yet been finally defined, a quick analysis considering only dimensional tolerances is sufficient. Thus, tolerance analyses with a low number of samples and correspondingly short computing times are suitable for a first qualitative estimation of the system behavior. For quantitative statements an increasing number of samples improves the quality of results. However, considering the case study, the determined standard deviation of the FKCs changes by a maximum of 4 % when the sampling number is increased from 5000 to 100000. Thus 5000 samples lead to sufficiently accurate results at the preliminary design stage.

Although the CAD skeleton model can be designed in any level of detail, it is useful to limit to the essential tolerances in terms of computing time and effort for the model set-up. Considering the degrees of freedom helps the designer to select deviations that need to be taken into account for the CAD-based tolerance analysis. For example, the consideration of form deviations is not useful for the case study.

Since the vector-based approach has already been proved to be appropriate for the analysis of skeleton models, the results of the case study are verified with the conventional approach. The resulting values for the FKCs of both approaches are practically identical and have a maximal average absolute deviation of  $1.2 \cdot 10^{-3}$ . These differences are mainly caused by the CAD-internal accuracy. Thus, the CAD-based approach is proved to be valid for the tolerance analysis of skeleton models. Due to the similarity of the representation forms of both approaches, this is in line with the expectations. However, since the vector representation of the skeleton of the scissor lift table leads to a non-linear system of equations and requires the generation of a plane equation, the vector definition of this model is complex and time-consuming.

## DISCUSSION AND OUTLOOK

Summing up, the proposed CAD-based approach supports the designer in the statistical tolerance analysis of skeleton models in preliminary design stages. The main benefit of this approach is that the designer themselves can easily build valid tolerance analysis models in the familiar CAD system. This is achieved by the fact that the geometry and especially the constraints already defined in the skeleton model are directly taken over for the tolerance analysis. Furthermore, the visual representation reduces the model's susceptibility to errors.

However, the CAD-based approach is particularly suitable for a quantitative evaluation of the robustness of the concept and a first validation of the function in preliminary design stages. Compared to the vector-based approach, the high computing time, which becomes relevant for detailed models and high sampling numbers, limits the application of this approach to simple geometries such as skeletons. However, if topological

changes (e.g. due to changing mating constraints) are neglected, the computing time can be significantly reduced. Instead of a multiple regeneration of the instance of the family tables, modern CAD systems, such as Creo Parametric 4.0<sup>®</sup>, offer the possibility of a quick verification of the instances without the need to open them in the front-end.

Furthermore, analogous to the vector-based approach, the creation and parametrization of tolerance zones can be challenging for a common designer. Since some deviations cannot be sufficiently mapped due to missing geometry elements in the skeleton model, the designer partially has to consider how these deviations affect the corresponding component (see tilt angle of legs of the case study). Although this step is challenging, it significantly contributes to the designer's improved understanding of the system and an increased sensitivity to tolerances. In addition to the training effect for the designer, the consistent application of the proposed approach ensures that a valid CAD skeleton model is already available in late preliminary design stages without the use of expensive CAT software. Thus, subsequent simulations (e.g. kinematic) provide reliable information about the final product already at the beginning of the detail design phase.

To further improve the usability of the approach, an automated generation and parameterization of tolerance zones on the basis of previously assigned tolerances would be useful. Furthermore, a reduction of the computing time as well as the extension to a space claim model, used for defining the boundaries of design spaces, is beneficial. This would enable the early representation of further geometry elements, such as cylindrical faces, and thus expands the application area of the approach.

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